COHOMOLOGY AND DEFORMATIONS IN GRADED LIE ALGEBRAS[1]

ALBERT NIJENHUIS AND R. W. RICHARDSON, JR.

Introduction. In an address to the Society in 1962, one of the authors gave an outline of the similarities between the deformations of complex-analytic structures on compact manifolds on one hand, and the deformations of associative algebras on the other. The first theory had been stimulated in 1957 by a paper [7] by Nijenhuis-Frölicher and extensively developed in a series of papers by Kodaira-Spencer, Kodaira-Spencer-Nirenberg and Kuranishi; the second had just been initiated by Gerstenhaber [9]. While fine details were not available at that time, it seemed that graded Lie algebras were the common core of both theories. In particular, in both cases, the set of deformed structures is represented by the set of solutions of a certain *deformation equation* in graded Lie algebras. This observation was further elaborated in a Research Announcement [16] of the authors, in which the concept of algebraic graded Lie algebra was carefully defined, and in which applications to deformations of Lie algebras and to representations, extensions and homomorphisms of algebras were indicated.

The present paper gives a detailed discussion of the deformation equation in graded Lie algebras whose summands are finite-dimensional. The paper starts with a general discussion of graded Lie algebras including the case of characteristic 2, and leads to a general deformation theorem, which is the precise analogue of Kuranishi's local completeness theorem for complex analytic structures [13]. (A more recent proof of this theorem [14] uses methods closely related to those indicated in [16].)

The basic deformation theorems presented are 16.2, 18.1, 20.3, 22.1 and 23.4. The following is an outline in which a few of the more technical details have been deleted. We consider a graded Lie algebra (cf. 3.1) $E=\oplus_{n=0}^{\infty} E^n$ in which each E^n is finite-dimensional, and with

Based on an address (entitled *Derivations and structures*) by Professor Nijenhuis delivered before the Vancouver meeting of the Society on August 28, 1962 by invitation of the Committee to Select Hour Speakers for Annual and Summer Meetings; received by the editors August 27, 1965.

[1] Research supported by an Office of Naval Research contract at the University of Washington, a Fulbright grant at the University of Amsterdam, an N.S.F. Postdoctoral Fellowship at the Institute for Advanced Study and the Mathematical Institute at Oxford, and N.S.F. Grant GP 1905 at the University of Pennsylvania.