CONCERNING A CONJECTURE OF WHYBURN ON LIGHT OPEN MAPPINGS

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Introduction. Some important and fundamental theorems in complex analysis are simple consequences of theorems in the theory of light open mappings for 2-manifolds. This rather complete theory is largely the work of G. T. Whyburn [9], [10], [11]. One theorem which includes the well-known theorems of Darboux [4] and Stoïlow [8] is the following:

THEOREM (WHYBURN). *Suppose that f is a light open mapping of a disk A (topological 2-cell) onto a disk B such that* (a) $f(\text{Int } A) = \text{Int } B$ *and* (b) $f|\text{ Bd } A$ *is a homeomorphism of* Bd A *onto* Bd B. *Then f is a homeomorphism.*

In his paper [12], Whyburn has conjectured that if in the above theorem each of A and B is a topological n-cell, then f is a homeomorphism. This is an extremely difficult problem. One result of this announcement provides an affirmative answer for special cases of this conjecture. Church and Hemmingsen [1], [2], [3] have made significant contributions on related problems. Meisters and Olech [7] have some results for very special types of light open mappings; namely, either locally 1-1 maps or locally 1-1 maps except on discrete sets of a certain type.

Here, each mapping is continuous and each space is metric. A mapping f of a space X into a space Y is light iff $f^{-1}f(x)$ is totally disconnected for each x in X. And, f is open iff for each U open in X, $f(U)$ is open relative to $f(X)$.

Suppose that f is a light mapping of a space X into a space Y. We shall say that the *singular set* S_f of f is the set of points x in X such that f is not locally 1-1 at x; i.e., there is no set U open in X and containing x such that $f|\, U$ is 1-1. We consider here only mappings f which *preserve both the boundary and the interior of* X (both of which are assumed to be nonempty).

THEOREM 1. *Suppose that X is a compact subset of a metric space M,* Bd $X \neq 0$, Int $X \neq 0$, *and f is a light open mapping of X into M such that* (1) $f(\text{Int } X) = \text{Int } f(X)$, (2) $f(\text{Bd } X) = \text{Bd } f(X)$, (3) *the singular*

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