

BOOK REVIEWS

Abelian categories. An introduction to the theory of functors. By Peter Freyd. (Harper's Series in Modern Mathematics.) Harper & Row, New York, 1964. 11+164 pp. \$7.00.

A healthy new seed was planted some twenty odd years ago in the well fertilized soil of the mathematical periodical literature—the notion of a category. It sprouted, took root, flowered, attracted bees, and by now the landscape is dotted with its progeny. It is a beneficent plant: mathematical gardeners have come to appreciate its usefulness in holding down the topsoil and preventing dust storms; indeed, some half dozen books have appeared within the past dozen years putting it to this use. It is a beautiful plant too, whose rapid proliferation has produced many unique and exotic variants; but, perhaps because of its increasingly multiform variety, the book extolling all its loveliness has not yet been written. So it is that the book at hand dwells at length only upon abelian categories, whose features now seem the most regular and symmetrical, and whose uses are at this point the best known. What may be somewhat surprising, therefore, or even somewhat lamentable, is that these uses are discussed not at length but only in passing, in the introduction and in some of the exercises. The book devotes itself, instead, to the proof of a rather miraculous result—the so-called full embedding theorem—according to which every small abelian category admits a full, exact embedding into the full category of modules over a suitable ring.

But now comes an odd turn of events. From the full embedding theorem there follows a strong metatheorem, to which is due much of the importance of the full embedding theorem in applications of abelian categories. According to the metatheorem, an assertion about a diagram is valid in every abelian category as soon as it can be obtained in each category of modules by a diagram chasing argument that, on the basis of given exactness and commutativity conditions, supplies missing maps at required locations and establishes specified additional exactness and commutativity conditions. Such is the strength of this metatheorem that, in the words of the author, "Much of the theory *within* abelian categories is reduced to the theory of modules." Strange, double edged metatheorem. Surely this statement seems better calculated to defeat than to support the encouragement of further study of abelian categories. Surely, indeed; and yet, equally surely, the value of the two principal ideas behind the proof of the full embedding theorem must be great in proportion to the