THE COHOMOLOGY OF RESTRICTED LIE ALGEBRAS AND OF HOPF ALGEBRAS[1,2]

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Communicated by W. S. Massey, November 6, 1964

Introduction. In theory, the bar construction suffices to calculate the homology groups of an augmented algebra. In practice, the bar construction is generally too large (has too many generators) to allow computation of higher-dimensional homology groups. In this paper, we outline a procedure which simplifies the calculation of the homology and cohomology of Hopf algebras.

Let A be a (graded) Hopf algebra over a field K of characteristic p. Filter A by $F_qA = A$ if $q \geqq 0$ and $F_qA = (I(A))^{-q}$ if $q < 0$, where $I(A)$ is the augmentation ideal. The associated graded algebra E^0A, $E^0_{q,r}A = (F_qA/F_{q-1}A)_{q+r}$, is clearly a primitively generated (bigraded) Hopf algebra over K. By a theorem due to Milnor and Moore [**4**], this implies that E^0A is isomorphic to the universal enveloping algebra of its restricted Lie algebra of primitive elements if $p > 0$, and to the universal enveloping algebra of its Lie algebra of primitive elements if $p = 0$.

Our procedure is to calculate $H^*(A) = \mathrm{Ext}_A(K, K)$ by means of a spectral sequence passing from $H^*(E^0A)$ to $H^*(A)$. The fundamental result is the construction of a reasonably small canonical $V(L)$-free resolution of the ground field, where $V(L)$ is the universal enveloping algebra of a restricted Lie algebra L. We also obtain such a $U(L)$-free resolution, where $U(L)$ is the universal enveloping algebra of a Lie algebra L. These resolutions allow computation of the E_2 term of the cited spectral sequence.

The author would like to express his deep gratitude to J. C. Moore, who suggested this approach to the problem of calculating the cohomology of Hopf algebras.

STATEMENT OF RESULTS. We first state the existence theorem for the required spectral sequence. Let A be a filtered augmented graded algebra over a field K. Let M be a left A-module and filter M by $F_qM = (F_qA)M$. Then E^0M is a left E^0A-module. Suppose that for $N = A$ and $N = M$ we have $N = \lim \mathrm{inv}\, N/F_qN$ and N is of finite type as

[1] During the preparation of this paper, the author was partially supported by National Science Foundation grant number NSF-GP-1853.

[2] The work announced here is contained in the author's doctoral thesis, submitted to Princeton University.