TRANSLATION-INVARIANT CONES OF FUNCTIONS ON SEMI-SIMPLE LIE GROUPS

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Introduction. Originally, the phrase harmonic analysis had a function-theoretic meaning, referring to the decomposition of a function into exponentials. In the current interpretation, particularly in connection with noncommutative groups, the term refers not to functions but to representations, and harmonic analysis is regarded as part of the theory of group representations. This shift in interpretation was motivated by the L^2-theory for compact groups. The decomposition of the L^2-space of a noncommutative compact group analogous to the Fourier decomposition for the circle involves multidimensional subspaces, and, as a result, there is no longer a canonical choice of a basis for the L^2-space analogous to the set $\{e^{in\theta}\}$ for the circle. The subspaces, on the other hand, are canonically determined, and correspond to the various irreducible representations of the group. It therefore became natural to regard irreducible representations as the basic building blocks of the theory in the place of the exponential functions.

Our purpose here is to call attention to some examples in the theory of semi-simple noncompact Lie groups where the classical setup prevails. That is, we shall find a class of functions on these groups which appear to play a role similar to that played by the exponentials for the circle or the real line. In terms of these functions, a form of spectral synthesis will be valid. Namely, for certain translation-invariant classes of functions on the group, we shall find that each function of such a class admits a unique representation as a generalized linear combination of the "exponentials" in that class. Admittedly, this result corresponds to a relatively easy case of spectral synthesis for the line. However, it is hoped that by pursuing this analogy further, other fruitful applications may be found.

The prototype of the theorem we shall prove is a theorem of Choquet and Deny [3] for R^n (or any locally compact commutative group). Let μ be a positive bounded Borel measure on R^n that does

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