SOME RESULTS CONCERNING COMPLETELY 0-SIMPLE SEMIGROUPS

BY GÉRARD LALLEMENT AND MARIO PETRICH

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We follow the notation and terminology of [1]. A semigroup T with zero is said to be *0-rectangular* if it has the property: if all the products at the vertices of a closed polygonal line (with a finite number of vertices) of the multiplication table are all but one equal to a nonzero element m and the remaining product is not zero, then it is also equal to m. A *rectangular 0-band* is a Rees matrix semigroup with zero over the one-element group.

THEOREM 1. *Let* $S=\mathfrak{M}^0(G; I, \Lambda; P)$. *Then the following statements are equivalent*:

(a) $S \cong G\times E/G\times\{0\}$, *where* E *is a rectangular* 0-*band*;

(b) *there exist invertible matrices* U $(I\times I)$ *and* V $(\Lambda\times\Lambda)$ *such that* $Q=VPU$ *is a regular matrix all of whose nonzero entries are equal to* 1;

(c) *there exist mappings* $\alpha: I\rightarrow G$, $\beta: \Lambda\rightarrow G$ *such that* $p_{\lambda i}=\beta(\lambda)\alpha(i)$ *if* $p_{\lambda i}\neq 0$;

(d) S *is* 0-*rectangular*;

(e) *if* $p_{\lambda_1 i_1}$, $p_{\lambda_1 i_2}$, $p_{\lambda_2 i_2}, \cdots, p_{\lambda_n i_n}$, $p_{\lambda_n i_1}\neq 0$, *then* $p_{\lambda_1 i_1}^{-1}p_{\lambda_1 i_2}p_{\lambda_2 i_2}^{-1}\cdots p_{\lambda_n i_n}^{-1}p_{\lambda_n i_1}=1$;

(f) S *has a subsemigroup intersecting each* $\mathcal{H}$-*class of* S *in exactly one element.*

The semigroup in (f) need not be unique. We note that an analogous result is valid for completely simple semigroups (i.e., without zero); in such a case (b) and (c) remain essentially the same, (a) becomes $S\cong G\times E$, E is a rectangular band, in (d) "0-rectangular" is replaced by "rectangular," in (e) it suffices to take four entries of P at a time, and (f) states that idempotents form a semigroup (and thus in this case the semigroup in (f) is unique).

An ideal I of a semigroup T is said to be a *matrix ideal* of T if: for all $a, b, c\in T$, (a) $aTb\subseteq I$ implies $a\in I$ or $b\in I$, (b) $abc\in I$ implies $ab\in I$ or $bc\in I$.

THEOREM 2. *Let S be a semigroup with a completely* 0-*simple ideal M. In order that there exist an M-homomorphism of S onto M, it is necessary and sufficient that* (0) *be a matrix ideal of S, and the restriction to M of the finest congruence ρ on S, having* 0 *as one of its classes and such that S/ρ is a rectangular* 0-*band, coincides with the* $\mathcal{H}$-*equivalence on M.*