A PRIORI ESTIMATES IN SEVERAL COMPLEX VARIABLES[1]

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Classically there are two points of view in the study of global existence problems in the theory of functions of a complex variable. One is to piece together local solutions (such as power series), always staying within the category of holomorphic functions. This method seems to have been initiated by Weierstrass; in the theory of several complex variables it has been implemented by the study of cohomology with coefficients in the sheaf of germs of holomorphic functions (more generally in the sheaf of germs of holomorphic sections of vector bundles). The second approach is to view the Cauchy-Riemann equations as a linear operator on C^∞ functions and to study this operator as an operator in Hilbert space; which leads to the Dirichlet integral and this method was first exploited by Riemann. In the theory of several complex variables this approach has led to the theory of harmonic integrals, which have been developed and widely applied in the compact case and which have recently been extended to the noncompact case. It is this extension which is the main concern of the present lecture. For simplicity we will deal with functions on a domain $M \subset \mathbf{C}^n$, although the results carry over to forms with coefficients in holomorphic vector bundles on finite manifolds.

Let $z^1, \cdots, z^n$ be coordinates in $\mathbf{C}^n$ and let $x^k = \mathrm{Re}(z^k)$ and $y^k = \mathrm{Im}(z^k)$. Then if u is a differentiable function we define u_{z^k} and $u_{\bar z^k}$ by

$$u_{z^k} = \frac{1}{2}\left(\frac{\partial u}{\partial x^k} - \sqrt{(-1)}\,\frac{\partial u}{\partial y^k}\right)$$

and

$$u_{\bar z^k} = \frac{1}{2}\left(\frac{\partial u}{\partial x^k} + \sqrt{(-1)}\,\frac{\partial u}{\partial y^k}\right).$$

Thus a function is holomorphic if and only if $u_{\bar z^k} = 0$, $k = 1, \cdots, n$. Here we are concerned with inhomogeneous equations:

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