THE EQUIVALENCE OF THE ANNULUS CONJECTURE AND THE SLAB CONJECTURE[1]

BY CHARLES GREATHOUSE

Communicated by M. L. Curtis, May 29, 1964

In [1], the author showed that the Slab Conjecture implies the Annulus Conjecture.

The purpose of this paper is to show that the Annulus Conjecture implies the Slab Conjecture for $n>3$ and hence the two conjectures are equivalent for $n>3$.

R^n, S^n will denote n-space and the n-sphere, respectively. A k-manifold N is embedded in a locally flat manner in an n-manifold M provided each point of N has a neighborhood U in M such that $(U, U\cap N)\approx(R^n, R^k)$.

The Annulus Conjecture. Let S_1^{n-1}, S_2^{n-1} be disjoint locally flat $(n-1)$-spheres embedded in S^n and let M be the submanifold of S^n bounded by $S_1^{n-1}\cup S_2^{n-1}$. Then $M\approx S^{n-1}\times[0, 1]$.

The Slab Conjecture. Let R_1^{n-1}, R_2^{n-1} be disjoint locally flat $n-1$ spaces embedded as closed subsets of R^n and let M be the submanifold of R^n bounded by $R_1^{n-1}\cup R_2^{n-1}$. Then $M\approx R^{n-1}\times[0, 1]$.

THEOREM. *The Annulus Conjecture implies the Slab Conjecture for $n>3$.*

PROOF. Let R_1^{n-1}, R_2^{n-1} be disjoint locally flat $n-1$ spaces embedded as closed subsets of R^n, $n>3$, and let M be the submanifold of R^n bounded by $R_1^{n-1}\cup R_2^{n-1}$. Let $S^n=R^n\cup\{p\}$ be the one-point compactification of R^n and $S_i^{n-1}=R_i^{n-1}\cup\{p\}$ for $i=1, 2$. By the corollary to Theorem 2 of [2], S_i^{n-1} is flat for $i=1, 2$. Hence, we may assume that $S_1^{n-1}=S^{n-1}$, that S_2^{n-1} lies in the northern hemisphere of $S^n=$ the suspension of S^{n-1}, and that $S_1^{n-1}\cap S_2^{n-1}=\{p\}$.

Let B^{n-1} be the unit ball in $S_1^{n-1}=S^{n-1}$ with center p, $r=$ the south pole of S^n, $q=$ the midpoint of the line segment joining p to r in S^n, $L=$ the line segment joining p to q in S^n, and B_r^n, $B_q^n=$ the cones (n-balls) in S^n with bases B^{n-1} and cone points r, q respectively. (See Figure 1.) Now, let $S_3^{n-1}=[S_1^{n-1}\cup\dot{B}_q^n]-\operatorname{Int}(B^{n-1})$. Then S_3^{n-1} is a flat $n-1$ sphere in S^n and $S_3^{n-1}\cap S_2^{n-1}=\varnothing$. By the Annulus Conjecture, $M\cup B_q^n=A^n$ is an n-annulus. We will complete the proof by showing that $M\cup\{p\}$ is homeomorphic to the decomposition space A^n/L and applying Lemma 3 of [3].

[1] This work supported in part by NSF GP-211.