AN EXAMPLE OF SLOW DECAY OF THE SOLUTION OF THE INITIAL-BOUNDARY VALUE PROBLEM FOR THE WAVE EQUATION IN UNBOUNDED REGIONS

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Let $\mathfrak{D}$ be a domain in the 3-dimensional Euclidean space E_3 and let $\mathfrak{B}$ be its boundary. Consider the initial-boundary value problem for the wave equation

$$(1) \qquad \nabla^2 u - u_{tt} = 0, \qquad x \in \mathfrak{D}, \qquad t > 0,$$

$$(2) \qquad u(x, 0) = f(x), \qquad u_t(x, 0) = g(x), \qquad x \in \mathfrak{D},$$

$$(3) \qquad u(x, t) = 0, \qquad x \in \mathfrak{B}, \qquad t > 0,$$

where $x = (x_1, x_2, x_3)$ and f and g are functions defined in $\mathfrak{D}$. It is well known that if $\mathfrak{D} = E_3$ and the initial data f and g have compact support then, at each point x, the solution $u(x, t)$ is zero after a finite time.

Morawetz [1] showed that if $\mathfrak{D}$ is the exterior of a smooth bounded star-shaped body and the initial data have compact support then u at each fixed point decays at least as fast as t^{-1}. Zachmanoglou [2] showed that the result of Morawetz is true even when the boundary $\mathfrak{B}$ extends to infinity and the initial data do not have compact support but they satisfy certain conditions at infinity.

Lax and Phillips [3] showed that if $\mathfrak{D}$ is the region exterior to a finite number of finite bodies then, at each point, u goes to zero. They showed that this is also true when the Dirichlet boundary condition (3) is replaced by the Newmann boundary condition

$$(4) \qquad \frac{\partial u}{\partial n}(x, t) = 0, \qquad x \in \mathfrak{B},\ t > 0.$$

Lax, Morawetz and Phillips [4] combined the result of Morawetz with the methods of Lax and Phillips to show that u at each point decays exponentially when $\mathfrak{D}$ is the region exterior to a bounded smooth star-shaped body and the initial data have compact support. It is the purpose of this note to show that this result is not generally true when the boundary $\mathfrak{B}$ extends to infinity and has a corner, even though the complement of $\mathfrak{D}$ is star-shaped.

Let $\mathfrak{D}$ be a domain bounded by two planes intersecting at an angle