ON THE COMPACTIFICATION OF ARITHMETICALLY DEFINED QUOTIENTS OF BOUNDED SYMMETRIC DOMAINS

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In previous papers [13], [2], [16], [3], [11], the theory of automorphic functions for some classical discontinuous groups Γ, such as the Siegel or Hilbert-Siegel modular groups, acting on certain bounded symmetric domains X, has been developed through the construction of a natural compactification of X/Γ, which is a normal analytic space, projectively embeddable by means of automorphic forms. The purpose of this note is to announce similar results for general arithmetic groups, which include the earlier ones as special cases.

1. For algebraic groups, we follow the notation and conventions of [4], [5]. $G \subset \boldsymbol{GL}(m, \boldsymbol{C})$ will be a semisimple linear algebraic group defined over $\boldsymbol{Q}$, and, for every subring B of $\boldsymbol{C}$, we put as usual $G_B = G \cap \boldsymbol{GL}(m, B)$. We let Γ be an *arithmetic subgroup* of G, i.e., a subgroup of $G_{\boldsymbol{Q}}$ commensurable with the group $G_{\boldsymbol{Z}}$ of units of G. The group acts on the right, in a properly discontinuous manner, on the symmetric space $X = K \backslash G_{\boldsymbol{R}}$, where K is a maximal compact subgroup of $G_{\boldsymbol{R}}$. Here we assume X to be *hermitian symmetric*, and therefore [7] equivalent to a bounded symmetric domain. The quotient $V = X/\Gamma$ then carries a natural ringed structure with which it becomes an irreducible normal analytic space.

There is no loss in generality in assuming G to be connected and centerless, and we shall do so. Although this is not essential, we shall here for simplicity assume that G is *simple over* $\boldsymbol{Q}$, in other words, that it has no proper invariant subgroup $N \neq (e)$ defined over $\boldsymbol{Q}$. We are, of course, interested here only in the case where V is not compact; this means that $G_{\boldsymbol{R}}$ has no compact factor $\neq (e)$ and that G contains a torus $S \neq (e)$ which splits over $\boldsymbol{Q}$ [5].

2. **Rational boundary components.** The space X has a "natural" compactification, obtained by taking the closure $\overline{X}$ of X in the Harish-Chandra realization of X as a bounded symmetric domain [7], [8]. The boundary is then the union of finitely many orbits of $G_{\boldsymbol{R}}$, each of which has a fibration, whose fibers are locally closed

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