PERIODIC MAPS WHICH PRESERVE A COMPLEX STRUCTURE

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1. **Introduction.** A *weakly complex structure* for a differentiable manifold M is, roughly, a structure for the stable tangent bundle of M as a complex vector space bundle; a map $T: M \to M$ is *weakly complex* if the differential dT is stably complex linear. We consider weakly complex maps $T: M \to M$, periodic of period p (usually p is prime). We study such problems as the relationship between M, F and the normal bundle to F.

There are two basic technical tools. First we study the complex bordism groups $\mathfrak{U}_n(X)$ of a space X, as a generalized homology theory. For B_{Z_p} a classifying space for the group Z_p, $\mathfrak{U}_n(B_{Z_p})$ is identified with the bordism group of weakly complex maps $T: M \to M$ of prime period p, operating on a closed manifold M without fixed points. The second technical tool is the theory of G-bundles $E \to B$ where a compact Lie group H acts on E as a group of bundle maps.

This work is a supplement to our previous study of periodic maps [1], and the methods are a continuation of those. A sample of the results here were given in our Seattle lectures [2]; Zelle has also studied aspects of weakly complex actions in his thesis [6]. A full account of our results will appear later.

2. **The complex bordism groups.** Given a bundle ξ of real $2k$-planes over a space X, a complex prestructure for ξ is a map J mapping each fiber of ξ linearly into itself and having $J^2 = -1$. A *complex structure* for ξ is a homotopy class of such prestructures; denote by $C(\xi)$ the set of complex structures. Denote by kI the trivial k-plane bundle $R^k \times X \to X$. For X a finite dimensional CW complex and for ξ a bundle of real n-planes over X, a *weakly complex structure* for ξ is an element of $C((2k-n)I+\xi)$, $2k-2 \geqq \dim X$; in an appropriate sense, this is independent of k.

A *weakly complex manifold* is a pair consisting of a differentiable manifold M and a weakly complex structure on the tangent bundle of M. The boundary of a weakly complex manifold is weakly complex; each weakly complex manifold has a uniquely defined negative.

Given a pair (X, A) of spaces, consider all pairs (M, f) where M is a weakly complex compact n-manifold and where $f: (M, \partial M) \to (X, A)$. Two such, (M_1, f_1) and (M_2, f_2), are bordant if there exists