ERGODIC PROPERTIES OF ISOMETRIES IN L^p SPACES, $1<p<\infty$

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Let $X=[0, 1]$, $\mathcal{B}$ the σ-algebra of Lebesgue measurable sets of X and μ the Lebesgue measure. For $1\leqq p<\infty$ denote with $\mathcal{L}^p$ the vector space of all real-valued $\mathcal{B}$-measurable functions f on X for which $|f|^p$ is integrable and with $f\to\|f\|_p=(\int|f|^p d\mu)^{1/p}$ the corresponding seminorm on $\mathcal{L}^p$. Denote with $\mathcal{L}^\infty$ the vector space of all real-valued $\mathcal{B}$-measurable functions on X which are essentially bounded and with $f\to\|f\|_\infty$ the essential supremum seminorm on $\mathcal{L}^\infty$. For each $1\leqq q\leqq\infty$ denote with L^q the associated Banach space and with $f\to\tilde{f}$ the canonical mapping of $\mathcal{L}^q$ onto L^q. If $T: L^q\to L^q$ is a continuous linear operator and $\tilde{f}\in L^q$, we shall denote by Tf a representative of the class $T\tilde{f}$. We shall say that *the individual ergodic theorem holds for* T if for every $f\in\mathcal{L}^q$

$$\lim_{m\to\infty}\frac{f(x)+Tf(x)+\cdots+T^{m-1}f(x)}{m}$$

exists almost everywhere. We shall say that *the dominated ergodic theorem holds for* T if there is a constant $C>0$ such that for every $f\in\mathcal{L}^q$

$$\sup_{1\leqq m<\infty}\frac{|f+Tf+\cdots+T^{m-1}f|}{m}\in\mathcal{L}^q$$

and

$$\left\|\sup_{1\leqq m<\infty}\frac{|f+Tf+\cdots+T^{m-1}f|}{m}\right\|_q\leqq C\|f\|_q.$$

Let us recall that an *automorphism* is a bijective mapping $\tau: X\to X$ satisfying the following two conditions: (i) for every $E\in\mathcal{B}$, $\tau^{-1}(E)\in\mathcal{B}$ and $\tau(E)\in\mathcal{B}$; (ii) if $A\in\mathcal{B}$ and $\mu(A)=0$, then $\mu(\tau^{-1}(A))=\mu(\tau(A))=0$. Let $\mathcal{A}$ be the group of all automorphisms, e the unit element of $\mathcal{A}$ (i.e. the identity mapping of X). For $\tau_1\in\mathcal{A}$, $\tau_2\in\mathcal{A}$, write $\tau_1\equiv\tau_2$ if $\mu(\{x\mid\tau_1(x)\neq\tau_2(x)\})=0$; this defines an equivalence relation R in $\mathcal{A}$. Denote with $\tau\to\tilde{\tau}$ the canonical mapping of the group $\mathcal{A}$ onto the quotient group $\mathcal{A}/R$.

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