CLASSIFICATION OF OPERATORS BY MEANS OF THE OPERATIONAL CALCULUS[1]

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1. **Introduction.** Let $\boldsymbol{A}=\boldsymbol{A}(\Delta)$ be a topological algebra of complex valued functions defined on a subset Δ of the complex plane, with the usual operations. Suppose that $\boldsymbol{A}$ contains the restrictions to Δ of polynomials. Let $B(X)$ be the Banach algebra of all bounded linear operators on the Banach space X into itself. We say that an operator T is *of class* $\boldsymbol{A}$ (notation: $T\in(\boldsymbol{A})$) if there exists a continuous representation $f\rightarrow T(f)$ of $\boldsymbol{A}$ into $B(X)$ such that $T(1)=I$ and $T(z)=T$. Such a representation is called an $\boldsymbol{A}$-*operational calculus* for T. A class $(\boldsymbol{A})$ may be as wide as $B(X)$ (if $\boldsymbol{A}$ consists of all entire functions with the topology of uniform convergence on every compact), or as narrow as the class of hermitian operators with spectrum in a given compact Δ (if $\boldsymbol{A}=C(\Delta)$, $T(\cdot)$ is norm decreasing, and X is a Hilbert space). Related approaches are found in [**3**; **5**].

2. **Restrictions on $\boldsymbol{A}$.** Let $H(\Delta)$ denote the algebra of all complex valued functions which are locally holomorphic in a neighborhood of Δ, with the usual topology.

CONDITION 1. If $f\in H(\Omega)$ for a compact $\Omega\neq\emptyset$, then there exists $f_0\in\boldsymbol{A}(\Delta)$ such that $f_0=f$ on $\Delta\cap\Omega_0$, for some neighborhood Ω_0 of Ω.

This condition excludes in particular the noninteresting case $\boldsymbol{A}(\Delta)=H(\Delta)$. *We shall consider here only* $\Delta=\boldsymbol{R}$ (the real line) or $\Delta=\boldsymbol{C}$ (the complex plane), and assume that $\boldsymbol{A}_0=\{f\in\boldsymbol{A}\mid f \text{ has compact support}\}$ is dense in $\boldsymbol{A}$.

Fix $f\in\boldsymbol{A}_0$. If $g\in H(\operatorname{Spt} f)$, Condition 1 implies the existence of $g_0\in\boldsymbol{A}$ such that $g_0=g$ on $\operatorname{Spt} f$. The map $M_f\colon H(\operatorname{Spt} f)\rightarrow\boldsymbol{A}$ given by $M_f g=fg_0$ is well defined.

CONDITION 2. The map $M_f\colon H(\operatorname{Spt} f)\rightarrow\boldsymbol{A}$ is continuous, for each $f\in\boldsymbol{A}_0$. A topological algebra $\boldsymbol{A}$ as in §1 which satisfies also Conditions 1 and 2 is called a *basic algebra* (compare [**5**]). Example: C^n for $0\leqq n\leqq\infty$.

3. **Restrictions on $T(\cdot)$.**

CONDITION 3. $T(\cdot)$ *has compact support* (denoted by Σ). If $g\in H(\Sigma)$ and $g_0\in\boldsymbol{A}$ is such that $g_0=g$ in a neighborhood of Σ (cf. Condition 1),

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