THE GROUP OF HOMOTOPY EQUIVALENCES OF A SPACE[1]

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Communicated by Deane Montgomery, December 2, 1963

1. **Introduction.** Let $\mathcal{E}(X)$ denote the collection of homotopy classes of homotopy equivalences of a space X with itself. Composition of maps induces a group structure in $\mathcal{E}(X)$. From the point of view of categories $\mathcal{E}(X)$ is the group of equivalences of the object X in the category of spaces and homotopy classes of maps. Thus it is the homotopy analog of the automorphism group of a group and the group of homeomorphisms of a space.

In this note we present some theorems which relate properties of the homotopy groups of X to algebraic properties of $\mathcal{E}(X)$. In such a study one encounters the difficulties associated with the problem of composing homotopy classes of maps. In addition one can easily show that for any finite group T there exists a finite complex X such that $\mathcal{E}(X)$ contains T as a subgroup. Thus any group theoretic property which does not hold for all finite groups cannot be true for all groups $\mathcal{E}(X)$.

The hypotheses of all of our theorems are not intricate, and thus our results provide specific information on $\mathcal{E}(X)$ for many X. §2 contains theorems on the group of equivalences of any 1-connected finite complex, and §3 deals with associative H-spaces. At the end of each section we give a brief description of our methods. Details and applications will appear elsewhere.

Various results on $\mathcal{E}(X)$ have been obtained by Barcus-Barratt [**1**, §6], P. Olum (to appear) and D. W. Kahn (to appear). Furthermore, W. Shih [**5**] has constructed a spectral sequence for $\mathcal{E}(X)$.

We should like to thank R. P. Langlands for several discussions on Proposition 9.

2. **General theorems.** We consider only 1-connected spaces of the homotopy type of a CW-complex with finitely generated homotopy groups in all dimensions. Let $X^{(n)}$ be an nth Postnikov section of X. A straightforward obstruction argument yields

LEMMA 1. *If X is a finite* CW-*complex then* $\mathcal{E}(X) \approx \mathcal{E}(X^{(n)})$ *for all* $n > \dim X$.

[1] Research supported by the U. S. Air Force Office of Scientific Research and the National Science Foundation (NSF-GP779).