RESIDUAL NILPOTENCE AND RELATIONS IN FREE GROUPS

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Introduction. Relations between elements of a free group lead directly to identities for all groups, which are of considerable importance. In 1958 R. C. Lyndon [7] initiated a study of relations in free groups; in particular Lyndon proved that if g_1, g_2, g are elements of a free group and

$$g_1^2 g_2^2 = g^2,$$

then g_1, g_2, g generate a cyclic group. Since then a number of generalizations of this theorem have been obtained by E. Schenkman [10], John Stallings [12], and Gilbert Baumslag [1; 2]. The most recent result of this kind proved by M. P. Schützenberger [11] (cf. also R. C. Lyndon and M. P. Schützenberger [8]) and, independently, also by Arthur Steinberg [13], states that if

$$g_1^p g_2^q = g^r,$$

where now p, q and r are integers greater than 1, then again g_1, g_2, g generate a cyclic group.

Similarly, if instead

$$g_1^{-1} g_2^{-1} g_1 g_2 = g^r \qquad (r > 1),$$

then once more g_1, g_2, g generate a cyclic group (M. P. Schützenberger [11], Gilbert Baumslag [3], and A. Karass, W. Magnus and D. Solitar [6], and Arthur Steinberg [13]).

The purpose of this note is to announce the following theorem which contains both the aforementioned theorems as special cases.

THEOREM 1. *Let $w = w(x_1, x_2, \cdots, x_n)$ be an element of a free group F freely generated by $x_1, x_2, \cdots, x_n$ which is neither a proper power nor a primitive.*[2] *If $g_1, g_2, \cdots, g_n, g$ are elements of a free group connected by the relation*

$$w(g_1, g_2, \cdots, g_n) = g^m \qquad (m > 1),$$

then the rank of the group generated by $g_1, g_2, \cdots, g_n, g$ is at most $n-1$.

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[2] An element in a free group is termed primitive if it can be included in a set of free generators.