RESEARCH ANNOUNCEMENTS

The purpose of this department is to provide early announcement of significant new results, with some indications of proof. Although ordinarily a research announcement should be a brief summary of a paper to be published in full elsewhere, papers giving complete proofs of results of exceptional interest are also solicited.

ON FLOQUET'S THEOREM IN HILBERT SPACES

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We are concerned with the question of extending the validity of Floquet's Theorem on linear periodic differential equations to equations in a Hilbert space. This question was raised and discussed (for equations in a Banach space) in [2]. Further results for Banach spaces will appear elsewhere.

Specifically, we consider, in a real or complex Hilbert space X, the equation

$$U + AU = 0, \tag{1}$$

where A is an operator-valued, locally (Bochner) integrable function of the real variable t, periodic with period 1 (for the sake of normalization); we denote by U the unique operator-valued solution of (1) that satisfies $U(0) = I$, where I is the identity operator. We set $U_0 = U(1)$.

We say that there is a *Floquet representation of order m* (m a positive integer) if there exists an operator B such that the operator-valued function P defined by $P(t) = U(t)e^{tB}$ is periodic with period m. It is easy to see that this is equivalent to the existence of a logarithm of U_0^m [2, Lemma 2.1].

The classical Floquet Theorem states that, if X is finite-dimensional, there is a Floquet representation of order 1, if the space is complex, and of order at most 2, if the space is real (see, e.g., [1, pp. 78, 81, 106–107]). It was shown in [2, Example 2.1] that if X is infinite-dimensional there need not be any Floquet representation at all; in that example, the space is real or complex, and separable, and A is continuous and differs by as little as we please (uniformly) from a constant of norm $= \pi$, so that $\int_0^1 \|A(t)\| dt$ exceeds π by as little as we please. On the other hand, it was shown in [2, Theorem 2.1] (by Banach space methods) that if $\int_0^1 \|A(t)\| dt < \log 4$, there always exists a Floquet representation of order 1. We shall now almost fill this gap between log 4 and π: