AXISYMMETRIC POTENTIAL PROBLEMS SUGGESTED BY BIOLOGICAL CONSIDERATIONS[1]

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1. **Introduction.** Although axisymmetric potential theory is a well-developed subject with many applications to the physical sciences it is, perhaps, not fully appreciated that certain biological problems suggest the use of this theory. In particular the problem of steady-state diffusional flow through a cylindrical structure arises frequently. Not surprisingly, the physiological situation may provide motivation for solving problems and seeking techniques that are different from those arising from purely mathematical or physical considerations. In this paper, which will be reported in full elsewhere, we describe one such problem and outline its resolution.

2. **Formulation and solution.** The system (1) given below can be regarded as a model for water loss through a plant pore into dry air surrounding the plant. In this model water vapor is the solute and air is the solvent. Let (r, θ, x) be cylindrical coordinates. Let $a>0$ and $l\geqq 0$ be constants and put $\lambda = l/a$. Put $R_1^0 = \{0<r<a;\ -l<x<0\}$, $R_2^0 = \{0<r<a;\ x=0\}$ and $R_3^0 = \{r>0;\ x>0\}$ and $R^0 = R_1^0 \cup R_2^0 \cup R_3^0$. Let $G(r)$ be a given function such that $rG^2(r)$ is Lebesgue integrable on $0\leqq r \leqq a$. We seek $u(r, x)$ from the system (1) of equations (n denotes outward drawn normal):

$$\text{(1a)} \qquad r\frac{\partial^2 u}{\partial x^2} + r\frac{\partial^2 u}{\partial r^2} + \frac{\partial u}{\partial r} = 0, \qquad (r, x) \in R^0,$$

$$\text{(1b)} \qquad \lim_{\epsilon\to 0^+} \int_0^a (u(r, -l+\epsilon) - G(r))^2 r\,dr = 0,$$

$$\text{(1c)} \qquad u(r, x) = 0, \qquad x > 0 \text{ and } r + x = \infty,$$

$$\text{(1d)} \qquad \frac{\partial u}{\partial n} = 0, \qquad \begin{array}{lll} r = 0 & \text{and} & x > -l, \\ r = a & \text{and} & -l < x < 0, \\ r > a & \text{and} & x = 0. \end{array}$$

If α_k denotes the kth positive root of $J_1(r)$ we write the corresponding Dini expansion of $G(r)$ as $G(r) \sim \sum_0^\infty \mathfrak{g}_k \mathfrak{J}_0(\alpha_k r/a)$ where $\mathfrak{J}_0(\alpha_0 r/a)$

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