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UNIVERSITY OF MINNESOTA

AN INTEGRATION-BY-PARTS FORMULA[1]

BY J. S. MAC NERNEY

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In 1914, W. H. Young [4] introduced a modification of the Riemann-Stieltjes integral which, for functions F and G defined on the real line with G of bounded variation on each interval and F suitably restricted, yields an additive interval function:

$$(Y)\int_a^b F\cdot dG + (Y)\int_b^c F\cdot dG = (Y)\int_a^c F\cdot dG.$$

In 1959, T. H. Hildebrandt [1] published a study of a certain linear initial-value problem involving these Young integrals, which extended some of the earlier results of H. S. Wall and of the present author (see [2] for discussion and references). In 1962, there was discovered a connection between the Young integral and the interior integral as introduced by S. Pollard in 1920 [3], *viz.*, the systems

$$U(x) = U(c) + (Y)\int_c^x U\cdot dH \quad \text{and} \quad V(x) = V(c) + (I)\int_x^c dH\cdot V,$$

with H a function from the real line to a complete normed ring, are naturally adjoint to one another [2, p. 326]. Both integrals are to be interpreted as limits in the sense of successive refinements of subdivisions of the interval of integration.

Suppose each of F and G is a function from the real line to the complete normed ring N. If each of F and G is of bounded variation

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