DUALITY AND RADON TRANSFORM FOR SYMMETRIC SPACES

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1. **The dual space of a symmetric space.** Let S be a symmetric space (that is a Riemannian globally symmetric space), and let $I_0(S)$ denote the largest connected group of isometries of S in the compact open topology. It will always be assumed that S is of *the noncompact type*, that is $I_0(S)$ is semisimple and has no compact normal subgroup $\neq \{e\}$. Let l denote the rank of S; then S contains flat totally geodesic submanifolds of dimension l. These will be called *planes* in S.

Let o be any point in S, K the isotropy subgroup of $G=I_0(S)$ at o and $\mathfrak{k}_0$ and $\mathfrak{g}_0$ their respective Lie algebras. Let $\mathfrak{g}_0=\mathfrak{k}_0+\mathfrak{p}_0$ be the corresponding Cartan decomposition of $\mathfrak{g}_0$. Let E be any plane in S through o, $\mathfrak{a}_0$ the corresponding maximal abelian subspace of $\mathfrak{p}_0$ and A the subgroup $\exp(\mathfrak{a}_0)$ of G. Let C be any Weyl chamber in $\mathfrak{a}_0$. Then the dual space of $\mathfrak{a}_0$ can be ordered by calling a linear function λ on $\mathfrak{a}_0$ positive if $\lambda(H)>0$ for all $H\in C$. This ordering gives rise to an Iwasawa decomposition of G, $G=KAN$, where N is a connected nilpotent subgroup of G. It can for example be described by

$$N = \left\{ z \in G \;\middle|\; \lim_{t\to\infty} \exp(-tH) z \exp(tH) = e \right\},$$

H being an arbitrary fixed element in C. The group N depends on the triple (o, E, C). However, well-known conjugacy theorems show that if N' is the group defined by a different triple (o', E', C') then $N'=gNg^{-1}$ for some $g\in G$.

DEFINITION. A *horocycle* in S is an orbit of a subgroup of the form gNg^{-1}, g being any element in G.

Let $t\to\gamma(t)$ (t real) be any geodesic in S and put $T_t=s_{t/2}s_0$ where s_τ denotes the geodesic symmetry of S with respect to the point $\gamma(\tau)$. The elements of the one-parameter subgroup T_t (t real) are called *transvections* along γ. Two horocycles ξ_1, ξ_2 are called *parallel* if there exists a geodesic γ intersecting ξ_1 and ξ_2 under a right angle such that $T\cdot\xi_1=\xi_2$ for a suitable transvection T along γ. For each fixed $g\in G$, the orbits of the group gNg^{-1} form a parallel family of horocycles.

Let M and M', respectively, denote the centralizer and normalizer of A in K. The group $W=M'/M$, which is finite, is called the Weyl group.

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