ON THE THEORY OF AUTOMORPHIC FUNCTIONS AND THE PROBLEM OF MODULI[1]

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The subject of this presentation is the problem of moduli for certain special types of algebraic-geometrical objects and the connection of this problem with the theory of automorphic functions for certain arithmetically defined discontinuous groups acting on bounded domains in complex Euclidean space.

To begin with, the earliest such situation of nontrivial content was the theory of elliptic modular functions, connected with the theory of moduli of elliptic curves. This situation was subsequently generalized to the theory of Siegel's modular functions connected with the moduli of normally polarized Abelian varieties. The theory of moduli of algebraic curves or of compact Riemann surfaces of genus n, which has received independent development from the analytical point of view by Teichmüller [9], Ahlfors [1], and Bers [4], may also be extracted to a large degree from the theory of moduli of normally polarized Abelian varieties of dimension n, from among which, by certain algebraic-geometrical criteria due to Matsusaka [6], may be distinguished those which are the canonically polarized Jacobian varieties of curves of genus n. The essential point is that the Jacobian varieties form a Zariski-open subset of an algebraic subset of the normally polarized Abelian varieties [2]. (Of course here we should be careful to point out the distinction between results of a more algebraic nature on the theory of moduli, and the results discussed here which are of a transcendental nature, linking such a theory of moduli with the theory of automorphic functions of some discontinuous group.)

The fact that we speak only of the moduli of polarized Abelian varieties comes of course from the well-known phenomenon that if, in the space of $n \times 2n$ period matrices of complex, n-dimensional tori, we identify those points which correspond to complex analytically isomorphic tori, what we get is a space which is not even Hausdorff, not to speak of being a complex space or algebraic variety. A polarization for a complex torus T is a class consisting of all divisors D

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