RESEARCH PROBLEMS

6. Richard Bellman: *Asymptotic behavior of solutions of functional equations.*

Write

$$f_N(x) = \max_{\{x_i\}} \sum_{i=1}^{N} g_i(x_i) \tag{1}$$

where the maximum is taken over the region $x_i \geqq 0$, $\sum_{i=1}^{N} x_i = x$, with $x > 0$. Under what conditions on the sequence $\{g_i(x)\}$ can we assert that $f_N(x) \sim N\phi(x)$ as $N \to \infty$?

Using the functional equation technique of dynamic programming, we see that

$$f_N(x) = \max_{0 \leqq y \leqq x} [g_N(y) + f_{N-1}(x - y)]. \tag{2}$$

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7. Richard Bellman: *Existence of closed subsystems.*

Consider a system of N simultaneous differential equations of the form $dx_i/dt = g_i(x_1, x_2, \cdots, x_N)$, $x_i(0) = c_i$, where the g_i are polynomials in the components x_i or, more generally entire functions.

Under what conditions on the g_i do there exist functions $h_i(y_1, y_2, \cdots, y_k)$, $k < N$, entire as functions of the y_i, with the property that the functions of t defined by $f_i = h_i(y_1, y_2, \cdots, y_k)$, $i = 1, 2, \cdots, k$, satisfy a set of simultaneous differential equations of the form $df_i/dt = G_i(f_1, f_2, \cdots, f_k)$, where the G_i are entire functions of their arguments? When these new variables exist, how does one determine them?

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