BOOK REVIEW

Banach spaces of analytic functions. By Kenneth Hoffman. Prentice-Hall Series in Modern Analysis, Prentice-Hall, New York, 1962. 13+217 pp.

This is an attractive book. To bring together the most important facts about Hardy's H^p spaces (Definition: $f \in H^p$ if $f \in L^p(0, 2\pi)$ and all Fourier coefficients of negative index vanish) was itself a bright idea, and the idea has been carried through with spirit. The book falls naturally into three parts. Chapters I–III contain classical preliminaries. Chapters IV-IX present all the important "finished" results of the modern theory. And the final Chapter X, more than twice as long as any other, is devoted to recent work on the Banach algebra H^∞.

Each chapter of the middle part (with the exception of Chapter IX, which investigates three disjoint aspects of the Banach space H^p) is a tightly connected exposition centered about a single theorem. Since taken together these constitute the big theorems of the subject, they are worth citing in detail.

In Chapter IV (theorem of Szegö-Kolmogoroff-Kreĭn): Let f' be the (a.e.) derivative of the increasing function f. Then

$$\exp \frac{1}{2}\pi \int_{-\pi}^{\pi} \log f' = \inf \int |1 - g|^2 df,$$

with the inf taken over all trigonometric polynomials $g = \sum_{k \geq 1} a_k e^{ik\theta}$.

In Chapter V (theorem of Riesz-Herglotz-Nevanlinna): Identify $f \in H^1$ with the obvious analytic F in the open disk. Suppose $F(0) > 0$. Then $F = BSA$ uniquely, where $B(z)$ is a "Blaschke product"

$$= \prod \left(\frac{\bar{\alpha}_n}{|\alpha_n|} \frac{\alpha_n - z}{1 - \bar{\alpha}_n z} \right)$$

with $\sum (1 - |\alpha_n|) < \infty$, and $S(z)$ is a "singular function"

$$= \exp \int \frac{e^{i\theta} + z}{e^{i\theta} - z} dH(\theta)$$

with H decreasing and $H' = 0$ a.e., and $A(z)$ is an "outer function"

$$= \exp \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{e^{i\theta} + z}{e^{i\theta} - z} \log |f(\theta)| \, d\theta.$$

In Chapter VI (theorem of Beurling-Rudin): Let $\mathfrak{A}$ be the algebra of analytic functions in the disk with continuous boundary values.