# ON LINKED BINARY REPRESENTATIONS OF PAIRS OF INTEGERS: SOME THEOREMS OF THE ROMANOV TYPE[1]

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1. **Introduction.** Let us denote by $N$ the sequence $\{1, 2, 3, \cdots\}$, by $p$ a prime, by $(a, b)$ the greatest common divisor of $a$ and $b$, by $[a, b]$ the least common multiple of $a$ and $b$, by $\{*: \cdots\}$ resp. $A\{*: \cdots\}$ the set resp. number of $*$ with the properties $\cdots$, by $\mu$ the Moebius function, by $C$ an absolute positive constant and by $C(*)$ a positive constant depending on $*$ only.

Suppose $N_j \subset N$ $(j=1, 2, 3, 4)$ and denote by $y_1 \sim y_2$ an arbitrary relation (= linking) with $y_{1,2} \in N$. For instance, $[y_1 \sim y_2] := [(y_1, y_2) = 1]$ resp. $[y_1 \sim y_2] := [y_1 = y_2]$ can be considered a weak resp. strong linking. By a linked binary representation of a pair $m$, $n$ with $m \in N$ and $n \in N$ we mean a solution $x_1, x_2, x_3, x_4$ of the Diophantine system $x_1 + x_2 = m \wedge x_3 + x_4 = n \wedge x_j \in N_j$ $(j=1, 2, 3, 4) \wedge x_2 \sim x_4$. Various generalizations are obvious (more summands, triples, etc.). We do not intend to give a detailed and general study of the questions arising in this context. We rather prefer to investigate two special problems of this type with $\sim$ being $=$; they are inspired by the following two well-known results of Romanov:

$$E_a := \{m : m = p + v^a \wedge p \text{ prime} \wedge v \in N\} \qquad (1 < a \in N)$$

and

$$F_a := \{m : m = p + a^v \wedge p \text{ prime} \wedge v \in N\} \qquad (a \in N)$$

have positive asymptotic density [1, pp. 63–70].

2. **On Romanov's first theorem.** Generalizing the result for $E_a$, we show that the set $\{m, n : m = p_1 + v^a \wedge n = p_2 + v^a \wedge p_{1,2} \text{ prime} \wedge v \in N\}$, considered as a set of lattice points in the plane, has positive asymptotic density in the plane:

THEOREM 1. *For $1 < a \in N$ there exist constants $C_1(a)$ and $C_2(a)$ such that $x > C_1(a)$ implies*

$$A_1(x, a) := A\{m, n : m < x \wedge n < x \wedge m = p_1 + v^a \wedge n = p_2 + v^a \wedge p_{1,2} \text{ prime} \wedge v \in N\} > C_2(a)\, x^2.$$

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