ON THE CONSISTENCY OF QUANTUM FIELD THEORY[1]

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Since its inception some thirty years ago the theory of quantum fields has come to be recognized as a fundamental component in any comprehensive description of the microcosmic physical world. The outstanding successes of the theory in this role indicate that its central features will almost surely persist in some form in whatever revisions of our view of the world are required by the discovery of new evidence. Yet in spite of its successes, this theory has been marred by the presence of internal difficulties, and the stubborn persistence of these difficulties against the best efforts of two generations has led to the wide-spread belief that the theory must somehow be founded on incompatible assumptions.

The question of the consistency of quantum field theory can be restated in purely mathematical terms, and in this form it becomes susceptible to a rigorous mathematical analysis. Our purpose here is to describe briefly the form this question takes and to present a program for resolving it which seems to us to offer a chance of success.

We begin by recalling that quantum field theory assumes that the physical world is made up of elementary particles of various types, and that each type of particle is described by a *quantum field* ϕ satisfying at least the following conditions [3].

(1) *Quantum condition.* ϕ is a tempered distribution defined on the Lorentz space-time manifold with values in a ring of (unbounded) operators acting on a fixed Hilbert space $\mathcal{H}$.

(2) *Covariance condition.* ϕ transforms covariantly according to a given (physically admissible) unitary representation U of the underlying Lorentz group.

(3) *Local commutativity.* $[\phi(f), \phi(g)]_{\pm} = 0$ whenever the test functions f and g have simultaneous supports.[2]

(4) *Existence of a vacuum.* There exists a vector ω in $\mathcal{H}$, invariant under U, which lies in the domain of all polynomial combinations of the field operators.

Examples of fields satisfying these conditions are known; they are the so-called free fields. There is essentially one irreducible free field

[1] Operated with support from the U. S. Army, Navy, and Air Force.

[2] The choice of sign here is determined by U, via the well-known connection between spin and statistics.