TWO THEOREMS CONCERNING FUNCTIONS HOLOMORPHIC ON MULTIPLY CONNECTED DOMAINS

BY E. L. STOUT[1]

Communicated by Walter Rudin, March 25, 1963

1. Let Ω be a finitely connected plane domain whose boundary, $\partial\Omega$, consists of the circles $\Gamma_0, \Gamma_1, \cdots, \Gamma_n$. We assume Γ_j lies in the interior of Γ_0 for $j=1, 2, \cdots, n$. Let Δ_0 be the interior of Γ_0 and let Δ_j be the exterior of Γ_j, $j=1, 2, \cdots, n$. We then have $\Omega=\bigcap_{j=0}^{n}\Delta_j$. Let $H_\infty[\Omega]$ be the collection of all bounded holomorphic functions in Ω. We shall say that a set S of points of Ω is an interpolation set for Ω if given a bounded complex valued function w on S there is $f\in H_\infty[\Omega]$ such that $f(z)=w(z)$ for all $z\in S$. If $\{z_n\}_{n=1}^{\infty}$ is a sequence in Ω, without limit points in Ω, we write $\{z_n\}=S_0\cup S_1\cup \cdots \cup S_n$ where the S_j are pairwise disjoint and where the only limit points of S_j lie in Γ_j, $j=0, 1, \cdots, n$.

In the present note we sketch proofs for the following two theorems:

THEOREM A. *The sequence $\{z_n\}$ is an interpolation set for Ω if and only if each S_j is an interpolation set for the disc Δ_j.*

THEOREM B. *Let $f_1, f_2, \cdots, f_m$ be functions in $H_\infty[\Omega]$ such that $|f_1(z)|+|f_2(z)|+\cdots+|f_m(z)|\geqq\delta>0$ for all $z\in\Omega$. Then there exist functions $g_1, g_2, \cdots, g_m\in H_\infty[\Omega]$ such that $f_1g_1+f_2g_2+\cdots+f_mg_m=1$.*

L. Carleson [2] has established Theorem B in case Ω is the open unit disc. He has also proved [1] that the sequence $\{z_n\}_{n=1}^{\infty}$ is an interpolation sequence for the open unit disc if and only if there is a $\delta>0$ such that

$$\prod_{n\neq k}\left|\frac{z_n-z_k}{1-\bar{z}_nz_k}\right|>\delta$$

for $k=1, 2, 3, \cdots$. For a discussion and alternative proof see [**3**, pp. 194–208].

2. **Outline of the proof of Theorem A.** Let B_j be the Blaschke product associated with the disc Δ_j and the set of points S_j, $j=0, \cdots, n$. Note that there is an $\eta>0$ such that $|B_j(z)|>\eta$ for $z\in S_k$ if $k\neq j$.

[1] National Science Foundation Graduate Fellow.