LIPSCHITZ CLASSES OF FUNCTIONS AND DISTRIBUTIONS IN E_n

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The results summarized here are the principle results of the author's doctoral dissertation presented at the University of Chicago and written under the direction of E. M. Stein. These results will appear soon with proofs.

We consider properties of classes of functions and distributions which are characterized by various smoothness and differentiability conditions. Similar classes have been studied by many investigators in recent years. Those papers touching most closely on the results given here are cited throughout this announcement. Special attention, however, is directed to the thorough list of references given recently by Nikolskiĭ in [9].

Our methods are analogous to those of Hardy and Littlewood in their study of analytic and harmonic functions in the unit disc (see [6]) extended to the n-dimensional nonperiodic case by consideration of "Poisson integrals" of tempered distributions in the same spirit as Stein and Weiss [11], and Stein [10].

We shall consistently denote by x and h elements of E_n, and by y elements of the positive real axis. By $L_p(E_n) = L_p$ we mean the normed linear space of measurable functions $f(x)$ for which the norm

$$\|f(x)\|_p = \|f\|_p = \left[\int_{E_n} |f(x)|^p dx\right]^{1/p}$$

is finite $(1 \leqq p < \infty)$. Using

$$\|f(x)\|_\infty = \operatorname*{ess\,sup}_{x \in E_n} |f(x)|,$$

we define $L_\infty(E_n) = L_\infty$ analogously. We also need notation for some mixed norms. Suppose $f(x, h)$ is measurable in x and h. Then define

$$\|f(x, h)\|_{pq} = \left[\int_{h \in E_n} \|f(x, h)\|_p^q |h|^{-n} dh\right]^{1/q}, \qquad 1 \leqq q < \infty,$$

and

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