This follows from the continuity of F, as before. The proof of the theorem is complete.

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TRANSVERSALITY IN MANIFOLDS OF MAPPINGS[1]

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1. **Introduction.** Let X and Y be differentiable manifolds and $\mathcal{A}$ a space of mappings from X to Y. A common problem in differential topology is to approximate a mapping in $\mathcal{A}$ by another in $\mathcal{A}$ which is transversal to a given submanifold $W \subset Y$. Thus if $\mathcal{A}_{X,W}$ is the subspace of mappings transversal to W it is important to know if $\mathcal{A}_{X,W}$ is dense in $\mathcal{A}$. Some famous examples are the Whitney immersion and embedding theorems [8] and the Thom transversality theorem [4; 7]. In the next section we give sufficient conditions for density in case $\mathcal{A}$ is a Banach manifold. The proof of the density theorem is indicated in the third section, and in the final section the Thom transversality theorem is obtained as a corollary.

2. **Density theorems.** Throughout this section X will be a manifold with boundary, Y and Z manifolds, $W \subset Y$ a submanifold (W, Y, Z without boundary) all of class C^r, $r \geqq 1$, and modelled on Banach spaces (see [3] for definitions).

2.1. DEFINITION. A C^r mapping $f: X \to Y$ is *transversal to W at a point* $x \in X$ iff either $f(x) \notin W$, or $f(x) = w \in W$ and there exists a neighborhood U of $x \in X$ and a local chart (V, ψ) at $w \in Y$ such that

$$\psi: V \to \boldsymbol{E} \times \boldsymbol{F}: V \cap W \to \boldsymbol{E} \times 0,$$

$\pi_1 \circ \psi$ is a diffeomorphism of $V \cap W$ onto an open set of $\boldsymbol{E}$, and $\pi_2 \circ \psi \circ f | U$ is a submersion [3, p. 20], where $\pi_1: \boldsymbol{E} \times \boldsymbol{F} \to \boldsymbol{E}$ and

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