MARKOV PROCESSES AND MARTIN BOUNDARIES

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1. **Introduction.** The theory of Martin boundaries associated with Markov processes has been established for the two typical and simplest classes; Brownian motions [**2**; **6**] and Markov chains on a denumerable space [**3**; **5**; **9**]. The purpose of this note is to give some general conditions under which one can construct the Martin boundary and derive the Martin representation of excessive functions,[1] generalizing the method in [**6**] and [**9**]. Either of our conditions ((C), (D) or (E)) is satisfied, for example, by sufficiently wide classes of diffusion processes and space-time processes as well as by the two classes cited above and so our results can be connected with and applied to some subjects in analysis such as differential equations or convolution transforms.

Two different cases are discussed separately. One of them (§3) is for the class of Markov processes such that there is a potential kernel of function type. In this case we shall also show a method of determining the potential kernel of function type which is somewhat different from Hunt's method in [**4**, III]. The other case (§4) is for the class of regular step processes [**10**] (including Markov chains) with the step measures having density functions. A full proof will appear elsewhere.

2. **Definitions and notations.** Let S be a locally compact, noncompact, separable Hausdorff space and X, a temporally homogeneous Markov process on S satisfying Hunt's condition (A) [**4**, I, pp. 48–50]. Following [**1**], such X is called a *Hunt process*. For the details of the definition see [**1**; **4**; **7**]. Let x_t denote the path functions of X, P_x and E_x the probabilities and expectations for X starting at x and σ_A the hitting time for a subset A of S, inf $\{t>0,\ x_t\in A\}$. We shall always assume that X is transient: $P_x\ (\sigma_{K^c}<\infty)=1$ for every x and every compact K of S, where K^c means the complement of the set K. For a measurable function u defined on S, $E_x(u(x_t))$ and $E_x(u(x_{\sigma_A}))$ are denoted by $H_tu(x)$ and $H_Au(x)$ respectively. We shall say u is *superharmonic* if it is positive[2] and if it satisfies $u(x)\geqq H_{A^c}u(x)$

[1] Meyer [**7**, Part II] also discussed this problem without introducing the boundary. His approach is based on Choquet's representation theorem for compact convex sets.

[2] The word 'positive' is used in the sense of 'non-negative.'