THE FUNDAMENTAL GROUP AND THE FIRST COHOMOLOGY GROUP OF A MINIMAL SET

BY HSIN CHU AND MICHAEL A. GERAGHTY[1]

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1. **Introduction.** A conjecture of long standing, posed by Professor W. H. Gottschalk, is whether an n-sphere cannot be a minimal set under a continuous flow, for an odd n greater than one. More generally, must a compact manifold which is minimal under a continuous flow have a nontrivial fundamental group. Or more generally yet, must a compact Hausdorff space which is minimal under a continuous flow have a nontrivial first integral cohomology group in the sense of Alexander-Wallace-Spanier.

Now let X be a locally pathwise-connected compact Hausdorff space such that every map of X into S^1 is homotopic to a constant, i.e. $\pi(X, S^1)=0$. Then it is shown in this paper that if X is minimal under a continuous flow, X must be totally minimal. The above questions then reduce to the existence of totally minimal flows.

It is further shown that for such spaces X a totally minimal flow cannot be locally almost periodic. So on such spaces one cannot have a locally almost periodic minimal flow.

In particular, for a sphere or real projective space of odd dimension greater than one or for a lens space, if it is a minimal set under a continuous flow, then it is totally minimal and so it is not locally almost periodic. For terminology we refer to Gottschalk-Hedlund [4]. Incidentally, the above results constitute a partial answer to Problem 1 of [5].

2. **The main theorem.** In the case of compact Hausdorff spaces X, it is known that $\pi^1(X)$ the Bruschlinsky group, which as a set is just $\pi(X, S^1)$, is isomorphic to the first integral A-W-S cohomology group $H^1(X)$. (See [6].) So either of these groups being zero implies that $\pi(X, S^1)$ is zero and every map of X to S^1 induces the zero homomorphism on $\pi_1(X)$. We may also derive this conclusion from the assumption that $\pi_1(X)$ has no factor group isomorphic with the integers.

THEOREM. *Let X be a compact, Hausdorff, locally pathwise-connected space such that for any map f from X to S^1, the induced homomorphism f_* on $\pi_1(X)$ is trivial. Then if X is a minimal set under a continuous flow, X is totally minimal.*

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