BOOK REVIEWS

Lie algebras. By Nathan Jacobson. Interscience Tracts on Pure and Applied Mathematics, no. 10. Interscience Publishers, New York, 1962. 9+331 pp. \$10.50.

Although Lie algebras are about 80 years old one might say that they are now barely past their adolescence. They appeared originally as "infinitesimal groups," i.e., as objects derived from Lie groups for the purpose of linearizing some of the basic group theoretical problems. However, it soon became evident that the resulting linear problems concerning the structure and the representations of Lie algebras were difficult and of a quite novel type, so that the algebraic technique existing at the turn of the century was inadequate to the tasks thus imposed on it.

It was this very difficulty which led to some of the most beautiful and most fruitful developments in linear algebra, the decisive steps being taken (in this order) by W. Killing, E. Cartan and H. Weyl (1888–1926). Since the methods used involved analysis and topology, it took several decades more—right up to the present—to put Lie algebra theory on a firm purely algebraic foundation and to extend the results to base fields other than that of the real or complex numbers. It is in this phase of Lie algebra theory that the author of the present book has played an important role. What he presents here is the first essentially complete and selfcontained exposition of the main results on the structure and the representations of Lie algebras which, moreover, embodies many original and substantial simplifications of the theory.

There are ten chapters, each of which begins with a brief description of its content and ends with a series of exercises designed so as to either supplement the theory in certain points or to develop some of the requisite algebraic technique. Knowledge of elementary linear algebra is presupposed and is a sufficient prerequisite for the study of all but the last chapter, where Galois theory and some of the Wedderburn structure theory of associative algebras is needed in addition.

It is a common feature of all existing structure and representation theory of algebras and groups that considerable heights have been attained for semisimple algebras and groups, while—in the general case—not much more has been achieved than the splitting off of the semisimple part from the radical and the superficial control of the solvable case. In this book, the main peaks of the semisimple theory are reached in Chapters IV and VII. Here, it is assumed that the base