ON THE EMERGENCE OF PATTERNS OF ORDER

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1. Introduction. The lecture which I have been asked to give is meant to honor a scientist whose work has encompassed a wide domain of knowledge. I have thought that a theme for tonight could be found in a review of some aspects of regularity and order as they appear in systems considered in mathematical physics. As this presents a very wide subject, I can do no more than touch upon a few examples.

Forms of order constitute the most basic fact which faces us in the phenomena of nature, and to become aware of a pattern of regularity has always been a striking experience. It has often evoked an attitude of respect for the forces of nature, and it has never ceased to tempt the mind to speculate what causes there may be behind the regularity. Even the question: Has this order a specific meaning? Does it point to something? has often turned up. Such a question, however, reaches beyond the realm of pure science, since any arrangement can be considered as representing some form of order when we take the mathematical point of view, and no form of order can then be said to be more important or to have more meaning than any other form.

It may be appropriate to review briefly certain cases of appearance of order and to focus attention upon the features from which they derive. Although this will not lead to new results, a kind of panorama outlook can have an attraction for itself. It is interesting to go over some of the steps which mark their explanation and to collect the essential points as far as possible into a single picture.

I will try to achieve this by describing a few examples of patterns in spatial arrangement. The first example will refer to crystal structure; the second one to a case of fluid motion; finally I will consider a mathematical equation describing an extremely simplified case of gas motion, under such circumstances that no pressure arises. For lack of time I leave aside phenomena of order in temporal sequences and shall not speak of periodic motions, etc.

When we start from the usual conceptions of Euclidean space in which, notwithstanding relativity and gravitation, most phenomena are still described, we are faced with its continuity and homogeneity. A primary requirement for the appearance of a pattern in such a

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