ALGEBRAS OF DIFFERENTIABLE FUNCTIONS

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1. Classification of certain spaces of continuously differentiable functions of two variables. Denote by C_0 the space of all complex-valued continuous functions on the plane that are zero at infinity. Write $\|\cdot\|_\infty$ for the supremum norm on C_0. Denote by D the dense subspace of C_0 consisting of infinitely differentiable functions with compact support.

Throughout we shall be concerned with differential operators of the form

$$\sum a_{m,n} \frac{\partial^{m+n}}{\partial x^m \partial y^n}; \tag{1.1}$$

the $a_{m,n}$ are complex constants. For each set $\mathfrak{A}$ of such operators, we define $C_0(\mathfrak{A})$ to be the space of all f in C_0 having Af in C_0 (in the sense of Laurent Schwartz) for all A in $\mathfrak{A}$. Equivalently, $C_0(\mathfrak{A})$ is the completion of D under the seminorms

$$f \to \|f\|_\infty \quad \text{and} \quad f \to \|Af\|_\infty, \qquad A \text{ in } \mathfrak{A}.$$

Each $C_0(\mathfrak{A})$ so defined is a translation-invariant space of functions; those that are furthermore invariant under rotations of the plane will be called *rotating spaces of differentiable functions.*

Certain of these spaces are familiar, namely the spaces C_0^N consisting of those functions in C_0 that have all derivatives of order $\leqq N$ in C_0, and the space C_0^∞, which is $\bigcap_N C_0^N$. A rotating space of differentiable functions will be called *proper* if it is not one of the C_0^N and not C_0^∞. Here is the classification of such spaces.

We use the notation

$$\frac{\partial}{\partial z} = \frac{1}{2}\left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right) \quad \text{and} \quad \frac{\partial}{\partial \bar{z}} = \frac{1}{2}\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right).$$

THEOREM 1.1. *If $\mathfrak{A}$ is a proper subset of*

$$\left\{\partial^{m+n}/\partial z^m \partial \bar{z}^n : m + n = N\right\}, \tag{1.2}$$

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