OPERATOR AVERAGES[1]

BY R. V. CHACON

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The object of this note is to state a general result the proof of which appears in [4] and which includes the ergodic theorem proved in [1] as well as the Hopf-Dunford-Schwartz ergodic theorem. Let $(S, \mathfrak{F}, \mu)$ be a σ-finite measure space, that is, let S be a set of points s, $\mathfrak{F}$ a Borel field of subsets A of S and μ a σ-finite measure defined on $\mathfrak{F}$. Let L_1 be the Banach space of complex-valued integrable functions $f(s)$ having S for their domain of definition. Dunford and Schwartz [5] have extended Hopf's ergodic theorem [6] as follows:

THEOREM. *Let T be a linear operator of L_1 to L_1 with $\|T\| \leqq 1$, and with* $(\|T\|_\infty \leqq 1)$

$$\operatorname*{ess.\,sup.}_{s\in S} | Tg(s) | \leqq \operatorname*{ess.\,sup.}_{s\in S} | g(s) | , \textit{ each } g \in L_1 \cap L_\infty.$$

Then

$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=0}^{n-1} T^k f(s)$$

exists almost everywhere, for f in L_1.

Ornstein and the author [1] proved the following theorem which was conjectured by Hopf [6]:

THEOREM. *Let T be a linear operator of L_1 to L_1 with $\|T\| \leqq 1$ and with $T \geqq 0$. Then*

$$\lim_{n\to\infty} \frac{\sum_{k=0}^{n} T^k f(s)}{\sum_{k=0}^{n} T^k p(s)}$$

exists almost everywhere on $\{s: 0 < \sum_{k=0}^{\infty} T^k p(s) \leqq +\infty\}$ for f and p in L_1, $p(s) \geqq 0$ almost everywhere.

It is of course clear that neither result contains the other and that

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