RESEARCH ANNOUNCEMENTS

The purpose of this department is to provide early announcement of significant new results, with some indications of proof. Although ordinarily a research announcement should be a brief summary of a paper to be published in full elsewhere, papers giving complete proofs of results of exceptional interest are also solicited.

INVARIANT QUADRATIC DIFFERENTIALS[1]

BY JOSEPH LEWITTES

Communicated by R. Bott, March 21, 1962

Let S be a compact Riemann surface of genus $g \geqq 2$ and h an automorphism (conformal homeomorphism onto itself) of S. h generates a cyclic group $H = \{I, h, \cdots, h^{N-1}\}$ where N is the order of h. We shall assume that N is a prime number. Let D_m for an integer $m \geqq 0$ denote the space of meromorphic differentials on S and $A_m \subset D_m$ the subspace of finite analytic (without poles) differentials. We obtain representations of H by assigning to h the linear transformation of D_m into itself by $h(\theta) = \theta h^{-1}$ for every $\theta \in D_m$. It is clear that h takes A_m into itself so that by restricting to A_m we have a representation of H by a group of linear transformations of a finite dimensional vector space.

In this note we are concerned with determining some of the properties of (h), the diagonal matrix for h, considering h as a linear transformation on the $3g-3$ dimensional space A_2 of quadratic differentials. Since $(h)^N = (I)$ it is clear that each diagonal element of (h) is an Nth root of unity. If $\epsilon \neq 1$ is an Nth root of unity, denote by n_k the multiplicity of ϵ^k $(k=0, 1, \cdots, N-1)$ in (h).

Let $\hat{S} = S/H$ be the orbit space of S under H. Then it is well known that $\hat{S}$ can be given a conformal structure and the projection map $\pi\colon S \to \hat{S}$ is then analytic. The branch points of this covering are precisely at the t fixed points of h, $P_1, \cdots, P_t \in S$, $t \geqq 0$—here we make essential use of the assumption that N prime—each a branch point of order $N-1$. Let g_1 be the genus of $\hat{S}$. The Riemann-Hurwitz formula reads $2g-2 = N(2g_1-2)+(N-1)t$. Now clearly n_0 is the dimension of that subspace of A_2 which consists of H—invariant differentials, i.e., those satisfying $h(\theta) = \theta$.

THEOREM 1. (i) *n_0, the dimension of the space of H-invariant finite quadratic differentials, is* $3g_1-3+t$.

(ii) *If $n_k \neq 0$ for some k, $1 \leqq k \leqq N-1$, then*

[1] This is a brief edited excerpt from my thesis submitted to Yeshiva University, 1962.