SOME RECENT DEVELOPMENTS IN THE THEORY OF PARTIAL DIFFERENTIAL EQUATIONS

CHARLES B. MORREY, JR.

1. **Introduction.** When I sent in the title of this talk, I thought it appropriate for me, as the first speaker in this general session of the Society, to make some general remarks about the whole field of partial differential equations for the benefit of those who are not familiar with the field. I soon found out that a comprehensive survey of the field would not be possible in 35 minutes. A glance at a few sections in Mathematical Reviews on this subject will indicate why.

Therefore what I shall do in this lecture is to speak briefly on a few selected problems of general interest with which I have not had much direct contact and then give more and more details about those problems in elliptic differential equations and the calculus of variations with which I am more familiar. Thus, I shall say very little about hyperbolic, parabolic, and unclassified equations, other than to define them in the next section. I give the following justifications for these omissions: (a) lack of time, (b) the fact that many of the methods and ideas of functional analysis, etc., which I shall illustrate in connection with elliptic equations also apply to hyperbolic and parabolic ones, and (c) there are extensive treatments of hyperbolic equations by Leray [36] and Gårding [25] who are now jointly writing a book on that subject; also there is a self-contained set of lecture notes by Yosida [93] which exploits the many connections between parabolic equations and the theory of semi-groups.

During the past twelve years, there has been increasing activity in the writing of books and sets of notes on this subject, beginning with the 1950 book by I. G. Petrowsky, a translation of which was published in 1954 [72]. Since then the books by Miranda (elliptic type only) [42], Duff [20], Rosenbloom [75], and J. L. Lions [40] have appeared; that by Rosenbloom contains 734 references, about 1/3 of which are to papers published between 1953 and 1957! Besides these, I know of books which are now being written by Hörmander, Browder, and myself which should appear shortly. No doubt there will be some small overlap between these books but, taken together, they should cover a large part of the field except possibly for applied mathematics and computation. It is regrettable that I shall not be

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