ON GROUPS WITH FINITELY MANY INDECOMPOSABLE INTEGRAL REPRESENTATIONS

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1. **Introduction.** The purpose of this note is to sketch a proof of the following theorem.

THEOREM. *If G is a finite group having finitely many non-isomorphic indecomposable integral representations then for no prime p does p^3 divide the order of G.*

It is known that the same hypothesis implies that all the Sylow subgroups of G are cyclic; thus they are cyclic of order p or p^2. We do not know whether the converse is true. On the other hand, we have shown elsewhere [1] that a cyclic group of order p^2 has finitely many non-isomorphic integral representations.

In the same place it is shown that the above theorem follows from this proposition:

PROPOSITION. *Let G be a cyclic group of order p^3. Then G has infinitely many non-isomorphic indecomposable representations over the p-adic integers.*

We outline below the proof of this proposition, which will appear in full elsewhere.

2. **Construction of indecomposables.** Let Λ be a ring such that the Krull-Schmidt theorem holds for finitely generated left Λ-modules; this is certainly the case for algebras of finite rank over a complete valuation ring [3]. We shall write Hom for Hom_Λ and Ext for Ext^1_Λ.

Suppose that M and N are indecomposable Λ-modules such that $\mathrm{Hom}(M, N)=0$, $\mathrm{Hom}(N, M)=0$. If $M^{(k)}$ is a direct sum of k copies of M then $\mathrm{Hom}(M^{(k)}, M^{(k)})$ may be identified with the ring of $k\times k$ matrices with entries in $H=\mathrm{Hom}(M, M)$. Also $\mathrm{Ext}(N^{(u)}, M^{(t)})$ consists of $t\times u$ matrices with entries in $\mathrm{Ext}(N, M)$. If $H'=\mathrm{Hom}(N, N)$ then $\mathrm{Ext}(N, M)$ is an (H, H')-bimodule, and $t\times t$ matrices over H and $u\times u$ matrices over H' operate in the obvious way on $\mathrm{Ext}(N^{(u)}, M^{(t)})$.

We shall say that a matrix $X\in\mathrm{Ext}(N^{(u)}, M^{(t)})$ is decomposable if there are invertible matrices T over H and U over H' such that

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