GENERALIZED MEASURES WHOSE VALUES ARE OPERATORS INTO AN INTERMEDIATE SPACE[1]

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The object of this note is the announcement of some results involving integration on $[-\infty, \infty]$ with respect to functions whose values are operators into an intermediate space (see Definition B). Besides several other function spaces of interest in analysis, Sobolev and Lebesgue spaces are particular types of intermediate spaces (cf., the recent contributions of CALDERÓN, GAGLIARDO, KREĬN, LIONS[2]); consequently, the present results extend previous ones dealing with Lebesgue spaces $L^p(a, \mathfrak{A}, \mu)$ (cf. KRABBE). Proofs and further theorems will appear elsewhere.

Each integrand will belong to the space G_0 of regulated functions[3] on the interval $(-\infty, \infty)$. Let $[G_0]$ consist of all $g \in G_0$ such that g coincides with its right-hand limit at each point; a Banach space $[G_0]$ is obtained by endowing $[G_0]$ with the topology of uniform convergence. Let G_1 be the family of functions of bounded variation on $(-\infty, \infty)$; in Definition C we shall characterize a strictly-decreasing sequence $\{G_\tau: 0 < \tau < 1\}$ of topological spaces such that

$$G_0 \supset \neq G_\iota \supset \neq G_\tau \supset \neq G_1 \qquad \text{(whenever } 0 < \iota < \tau < 1).$$

Let E be a linear subset of a Banach space $\mathfrak{Y}$. If $E|\mathfrak{Y}$ is the normed space obtained by endowing E with the norm of $\mathfrak{Y}$, then $[E, \mathfrak{Y}]$ will denote the Banach space[4] of all bounded linear mappings of $E|\mathfrak{Y}$ into $\mathfrak{Y}$ (if E is dense in $\mathfrak{Y}$, then $[E, \mathfrak{Y}]$ is identifiable with the Banach space $[\mathfrak{Y}, \mathfrak{Y}]$ of continuous linear operators in $\mathfrak{Y}$).

Our main result (Theorem B in §1) extends the following integral-representation property: if $g \rightarrow Ug$ is a bounded linear mapping of the Banach space $[G_0]$ into $[E, \mathfrak{Y}]$, then the Stieltjes sums corresponding to the integral

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[2] An author's name in small capitals indicates a reference to the bibliography.

[3] That is, G_0 consists of all the complex-valued functions g having only simple discontinuities on $(-\infty, \infty)$, and such that $|g(x+0)| \neq \infty$, $|g(\lambda-0)| \neq \infty$ whenever $-\infty \leqq x < \infty$ and $-\infty < \lambda \leqq \infty$.

[4] The topology of $[E, \mathfrak{Y}]$ is the uniform operator-topology generated by the usual operator-norm; see 1.3.