tion is assumed with respect to ϕ. The space would not be an Orlicz space, but an extension of the L_p space for $p<1$. For the latter L_p spaces, it is known that the isometries are as described above.

References

1. S. Banach, *Théorie des opérations linéaires*, Warsaw, Monogr. Mat., Tom 1, 1932.

2. R. V. Kadison, *Isometries of operator algebras*, Ann. of Math. vol. 54 (1951) pp. 325–338.

3. M. A. Krasnosel'ski and Ya. Ruticki, *Convex functions and Orlicz spaces* (in Russian), Moscow, Gosudarstv. Izdat. Fiz.-Mat. Lit., 1958.

4. J. Lamperti, *On the isometries of certain function spaces*, Pacific J. Math. vol. 8 (1958) pp. 459–466.

5. G. Lumer, *Semi-inner-product spaces*, Trans. Amer. Math. Soc. vol. 100 (1961) pp. 29–43.

6. ———, *Semi-inner-product spaces and isometries*, (to be published).

7. W. Orlicz, *Über eine gewisse Klasse von Räumen von Typus* B, Bull. Inst. Acad. Polon. Sci. Ser. A (1932) pp. 207–220.

8. M. H. Stone, *Applications of the theory of Boolean rings to general topology*, Trans. Amer. Math. Soc. vol. 41 (1937) pp. 375–481.

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ON THE RECURRENCE OF SUMS OF RANDOM VARIABLES

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We give a very short proof of the recurrence theorem of Chung and Fuchs [1] in one and two dimensions. This new elementary proof does not detract from the old one which uses a systematic method based on the characteristic function and yields a satisfactory general criterion. But the present method, besides its brevity, also throws light on the combinatorial structure of the problem.

Let N denote the set of positive integers, M that of positive real numbers. Let $\{X_n, n\in\mathrm{N}\}$ be a sequence of independent, identically distributed real-valued random vectors, and let $S_n=\sum_{\nu=1}^{n} X_\nu$. The value x is possible iff for every $\epsilon>0$ there exists an n such that $P\{|S_n-x|<\epsilon\}>0$; it is recurrent iff for every $\epsilon>0$, $P\{|S_n-x|<\epsilon$ for infinitely many $n\}=1$. It is shown in [1] that every possible value is recurrent if and only if for some $m\in\mathrm{M}$ we have

$$\sum_{n=1}^{\infty} P\{|S_n|<m\}=\infty. \tag{1}$$

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