THE GENERAL COEFFICIENT THEOREM AND CERTAIN APPLICATIONS[1]

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Teichmüller was the first person to point out explicitly the connection between quadratic differentials and the solutions of certain extremal problems in Function Theory. He enunciated the principle that if a point is required to be fixed the quadratic differential will have a simple pole there, if in addition fixed values are required for the first n derivatives of competing functions the quadratic differential will have a pole of order $n+1$. He was led to this principle by abstraction from the numerous results of Grötzsch and by his considerations on quasiconformal mapping. However, he never proved any general result embodying this principle.

The General Coefficient Theorem provides such a result and includes as special cases virtually every result in the theory of univalent functions. We now formulate it in the following form [**6**; **8**], more general than that of earlier statements [**1**; **2**].

GENERAL COEFFICIENT THEOREM. *Let $\mathfrak{R}$ be a finite oriented Riemann surface, $Q(z)dz^2$ a positive quadratic differential on $\mathfrak{R}$, $\{\Delta\}$ an admissible family of domains Δ_j, $j=1, \cdots, K$, on $\mathfrak{R}$ relative to $Q(z)dz^2$ and $\{f\}$ an admissible family of functions f_j, $j=1, \cdots, K$, associated with $\{\Delta\}$. Let $Q(z)dz^2$ have double poles $P_1, \cdots, P_r$ and poles $P_{r+1}, \cdots, P_n$ of order greater than two. We allow either of these sets to be void but not both. Let P_j, $j\leqq r$, lie in the domain Δ_l and in terms of a local parameter z representing P_j as the point at infinity let f_l have the expansion*

$$f_l(z) = a^{(j)}z + a_0^{(j)} + \textit{negative powers of } z \tag{1}$$

and Q the expansion

$$Q(z) = \alpha^{(j)}z^{-2} + \textit{higher powers of } z^{-1}. \tag{2}$$

Let P_j, $j>r$, a pole of order m_j greater than two lie in the domain Δ_l and in terms of a local parameter z representing P_j as the point at infinity let f_l have the expansion

An address delivered before the Chicago meeting of the Society on April 14, 1961 by invitation of the Committee to Select Hour Speakers for Western Sectional Meetings; received by the editors September 20, 1961.

[1] This address was delivered at a time when the speaker was receiving support from the National Science Foundation.