RESEARCH ANNOUNCEMENTS

The purpose of this department is to provide early announcement of significant new results, with some indications of proof. Although ordinarily a research announcement should be a brief summary of a paper to be published in full elsewhere, papers giving complete proofs of results of exceptional interest are also solicited.

COVERINGS IN FREE LATTICES

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The purpose of this note is to exhibit some new coverings in the free lattice, FL(n), generated by a finite number of elements. Free lattices were studied extensively by P. M. Whitman [1; 2] who found a number of coverings. Theorem 1 of this note guarantees an infinite number of distinct pairs of covering elements.

Let FL(n) have generators $x_1, \cdots, x_n$. The definition of the words (elements) of the lattice and the ordering of the words are those of Whitman [1]. We make use of his Lemma 1.1 appearing in [2]:

In FL(n), if w is any word and x_r is any generator, then either $w \geqq x_r$ or $\bigcup_{i \neq r} x_i \geqq w$, but not both.

LEMMA. *In FL(n) let $w \geqq x_r$. If $[w \cap (\bigcup_{i \neq r} x_i)] \cup x_r \geqq w$ then w covers $w \cap (\bigcup_{i \neq r} x_i)$.*

PROOF. First note that $w \neq w \cap (\bigcup_{i \neq r} x_i)$ as $w \geqq x_r$, but $\bigcup_{i \neq r} x_i \ngeqq x_r$. Now suppose that $w \geqq y \geqq w \cap (\bigcup_{i \neq r} x_i)$. If $y \ngeqq x_r$, then $\bigcup_{i \neq r} x_i \geqq y$ and hence $y = w \cap (\bigcup_{i \neq r} x_i)$. If $y \geqq x_r$, then $w \leqq [w \cap (\bigcup_{i \neq r} x_i)] \cup x_r \leqq y \cup x_r = y \leqq w$ and hence $y = w$.

THEOREM 1. *In FL(n), let $\bigcup_{i \neq r} x_i \geqq y$. Then $x_r \cup y$ covers $(x_r \cup y) \cap (\bigcup_{i \neq r} x_i)$.*

PROOF. We set $w = x_r \cup y$ and verify the criteria of the lemma. Since $\bigcup_{i \neq r} x_i \geqq y$ it follows that $(x_r \cup y) \cap (\bigcup_{i \neq r} x_i) \geqq y$ and hence $[(x_r \cup y) \cap (\bigcup_{i \neq r} x_i)] \cup x_r \geqq x_r \cup y$. The theorem now follows.

COROLLARY. *Let FL(3) have generators a, b, and c. If w is any word, then $a \cup (b \cap w)$ covers $[(a \cup (b \cap w)] \cap (b \cup c)$.*

PROOF. $b \cup c \geqq b \geqq b \cap w$.

Note that $a \cup (b \cap w)$ is the form of a typical element in an infinite ascending chain of elements in FL(3) as established in [2], thus this corollary gives an infinite number of pairs of distinct coverings in FL(3).