CARTESIAN PRODUCTS OF CONTRACTIBLE OPEN MANIFOLDS

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1. **Introduction.** In [6], J. H. C. Whitehead gave the first example of a contractible open 3-manifold W topologically different from E^3. Shapiro and Glimm [2] have shown that $W\times E^1$ is topologically E^4, and Glimm has also noted that $W\times W$ is topologially E^6. It is the purpose of this note to show that these results hold in general. The unsettled nature of the Poincaré Conjecture for dimension 3 dictates the assumption that each compact subset of the contractible open 3-manifolds considered can be embedded in the 3-sphere S^3. Such a 3-manifold will be called a *W-space*. The author does not know if each W-space can be embedded in S^3 (see §4).

The terminology is standard. An n-manifold is a countable, connected locally-finite simplicial complex such that the link of each vertex is piecewise-linearly homeomorphic to the usual $(n-1)$-sphere. "Open" is interpreted to mean "without boundary and noncompact," and "closed" to mean "without boundary and compact." All spaces and mappings are taken in the polyhedral or piecewise-linear sense unless stated otherwise. Finally, all the manifolds considered here will be orientable. The author wishes to thank R. H. Bing and David Gillman for their constructive suggestions concerning this paper.

2. **Constructing** W**-spaces.** All known examples of W-spaces can be expressed as the sum of a properly ascending sequence of cubes with handles (i.e., multiple solid tori). Theorem 1 shows this to be a general phenomenon. The author has investigated some examples of W-spaces topologically different from Whitehead's first example. In particular, the cubes with handles used in constructing a W-space may need to be a genus greater than 1 (as in [3]).

A 3-manifold M with nonempty boundary is *irreducible* if each loop in Bd M that can be shrunk to a point in M can also be shrunk to a point in Bd M (e.g., a cube with a knotted tubular hole is irreducible). Otherwise, M is *reducible.* The following is a consequence of the loop theorem and Dehn's lemma of Papakyriakopoulos or, diectly, of [5].

LEMMA 1. *M is reducible if and only if there is a disk D in M such that $D\cdot$*Bd *$M=$*Bd *D and* Bd *D does not bound a disk in* Bd *M.*