BOUNDARY VALUE PROBLEMS FOR PSEUDOANALYTIC FUNCTIONS

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The object of this note is the discussion of certain linear boundary value problems associated with the theory of linear elliptic systems of partial differential equations in two independent variables. Such problems were treated for multiply connected plane domains by I. N. Vekua [**10**], who successfully applied a technique utilizing singular integral equations. The results suggested the possibility of studying these problems for domains which are not schlicht. Unfortunately, the methods developed by Vekua did not lend themselves to generalization without scrutiny (see [**8**]). It appeared best, therefore, to restudy the question from an entirely different viewpoint. This involved an independent treatment of boundary value problems for analytic functions [**3**; **4**; **7**; **9**]. Below, we shall indicate how results for such problems in the theory of analytic functions may be transferred to results for corresponding problems in the theory of pseudoanalytic functions. Proofs and further theorems will appear elsewhere.

1. **Formulation of the Riemann-Hilbert problem.** Letting D denote a given Riemann surface of finite genus, with a boundary $\dot{D}$ consisting of a finite number of Liapounov curves, i.e., curves with Hölder continuously turning tangents, and interior D_0, we shall now pose a boundary value problem which we denote by RH(a, b, γ, Λ). Here Λ and γ are to be given functions on $\dot{D}$, with $|\Lambda| = 1$ and γ real. Furthermore, Λ is to be Hölder continuously differentiable, while γ is assumed to be Hölder continuous. a and b are given coefficients of a conjugate differential on D_0, i.e., $ad\bar{z}$ and $bd\bar{z}$ are invariant under conformal transformations. We assume here that a and b are bounded and measurable in the sense that if g is the coefficient of a nonvanishing, continuous differential on D, i.e., gdz is invariant under conformal transformations, then $a/\bar{g}$, $b/\bar{g}$ are bounded, measurable functions on D_0. The problem RH(a, b, γ, Λ) will consist of finding functions w continuous on D, such that

$$w_{\bar{z}} = aw + b\bar{w} \tag{1.1}$$

in D_0, and

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