EQUIVALENCE OF NEARBY DIFFERENTIABLE ACTIONS OF A COMPACT GROUP

BY RICHARD S. PALAIS[1]

Communicated by Deane Montgomery, April 5, 1961

In this note we will be concerned with the proof and consequences of the following fact: if ϕ_0 is a differentiable action of a compact Lie group on a compact differentiable manifold M, then any differentiable action of G on M sufficiently close to ϕ_0 in the C^1-topology is equivalent to ϕ_0.

1. **Notation.** In what follows differentiable means class C^∞. If M and V are differentiable manifolds, $\mathfrak{M}(M, V)$ is the space of differentiable maps of M into V in the C^K-topology where K is a positive integer or ∞ fixed throughout. We denote by Diff (M) the group of automorphisms of M topologized as a subspace of $\mathfrak{M}(M, M)$. As such it is a topological group. $\mathfrak{D}(M)$ is the subgroup of Diff (M) consisting of diffeomorphisms which are the identity outside of some compact set and $\mathfrak{D}_0(M)$ is the arc component of i_M, the identity map of M, in $\mathfrak{D}(M)$. If M is compact $\mathfrak{D}(M)$ is locally arcwise connected and $\mathfrak{D}_0(M)$ is open in $\mathfrak{D}(M)$ and in fact in $\mathfrak{M}(M, M)$. For a definition of the C^K-topology and a proof of the statements made above, see [6]. If G is a Lie group we denote by $\mathfrak{A}(G, M)$ the space of differentiable actions of G on M, i.e. continuous homomorphisms of G into Diff (M), topologized with the compact-open topology. If $\phi: g \to g^\phi$ is an element of $\mathfrak{A}(G, M)$ then by a theorem of D. Montgomery [2] $\tilde{\phi}: (g, m) \to g^\phi m$ is an element of $\mathfrak{M}(G \times M, M)$. Given $\phi \in \mathfrak{A}(G, M)$ and $f \in$ Diff (M) then ϕ composed with the inner automorphism of Diff (M) defined by f is another element $f\phi$ of $\mathfrak{A}(G, M)(g^{f\phi} = fg^\phi f^{-1})$. Clearly $(f, \phi) \to f\phi$ is jointly continuous[2] and defines an action of Diff (M) on $\mathfrak{A}(G, M)$. We henceforth consider $\mathfrak{A}(G, M)$ as a Diff (M)-space and, *a fortiori* as a $\mathfrak{D}(M)$ and $\mathfrak{D}_0(M)$-space. Note that the orbit space $\mathfrak{A}(G, M)/$Diff (M) is just the set of equivalence classes of actions of G on M.

2. **Statement of main theorem and consequences.** The following theorem will be proved in §3.

THEOREM A. *If M is a compact differentiable manifold and G is a compact Lie group then the $\mathfrak{D}_0(M)$-space $\mathfrak{A}(G, M)$ admits local cross sections; i.e. given $\phi_0 \in \mathfrak{A}(G, M)$ there is a neighborhood U of ϕ_0 in*

[1] Partially supported by NSF Grant No. G-14227.

[2] This follows from the proposition in [6, §1].