BOOK REVIEWS

THEOREM. Every commutative Moufang loop which can be generated by n elements, where $n \ge 3$, is centrally nilpotent of class at most n-1.

This result in fact is so recent that it does not appear elsewhere in the literature.

On the whole the book is highly original. Many of the published results appear in more lucid form. The book is also very readable and appears free of misprints worth mentioning. In other words what it chooses to treat is done in an excellent way. Without a doubt this book is a must for anyone even vaguely interested in the study of loops. To the beginner it offers a survey of the literature on binary systems coupled with an excellent bibliography. It is also a work that is likely to stimulate interest and further research in binary systems. ERWIN KLEINFELD

Foundations of geometry, Euclidean and Bolyai-Lobachevskian geometry, projective geometry. By K. Borsuk and Wanda Szmielew. Revised English translation. Amsterdam, North-Holland Publishing Company, 1960. \$12.00.

Euclid's long lasting popularity is readily explained by the lack of progress on the field covered by the *Elements* during more than twenty centuries although perhaps this sterility was merely a consequence of the authority imposed by the Greek geometrician. The same argument would not suffice to explain the unshaken confidence with which Hilbert's Grundlagen der Geometrie is still considered by many people as the definitive revelation of geometric truth. Too much has happened in the last sixty years in the axiomatics of geometry. When I had the opportunity to review the 8th edition of Hilbert's Grundlagen der Geometrie in Nieuw Archief voor Wiskunde. I tried to answer this question by a detailed analysis of the historical context of that work, and by an appraisal of its positive qualities as well as of its drawbacks. The still overpowering appeal of Hilbert's work is attested anew by the fact that two renowned Polish mathematicians have engaged in the difficult and not too grateful task of elaborating Hilbert's work and adapting it in detail to a more modern concept of mathematics. Though in Euclidean and Lobachevskian geometry Borsuk's and Szmielew's work covers only a small part of Hilbert's booklet, its extent is three or four times that of its predecessor. This proves anew, if ever proof was needed, that the brevity of Hilbert's work was bought by extensive, though mostly minor, omissions. It also proves that Hilbert's lay-out is too complicated and that it can hardly serve as a basis of a simple axiomatic introduction into geom-

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