BOOK REVIEWS

Lectures on the theory of integral equations. By I. G. Petrovskii. Trans. from the second (1951) Russian Edition by H. Kamel and H. Komm. Rochester, Graylock, 1957. vi+94 pp. \$2.95.

This monograph has been translated into very readable English by the translators and gives an excellent expository introduction to the theory of integral equations of the second kind:

$$\phi(P) = \lambda \int_G K(P, Q)\phi(Q)dQ + f(P) \tag{1}$$

where P, Q belong to G, a d-dimensional domain bounded by a finite number of pieces of $(d-1)$-dimensional surfaces and $f(P)$ is piecewise continuous. The parameter λ may or may not appear explicitly.

The elastic string is given as a motivating example. This is followed by approximating (1) by means of replacing the integral by finite Riemann sums. Solutions and conditions insuring solutions of this linear algebraic system are noted and, instead of passing to the limit, these algebraic results are used to suggest the Fredholm alternative theorems for the integral equation (1).

These theorems are then proved in turn for: (i) degenerate kernels $K(P, Q) = \sum_{i=1}^{m} a_i(P)b_i(Q)$, which reduce immediately to linear algebraic systems; (ii) sufficiently small uniformly continuous kernels, where (1) is solved by successive approximations obtaining a Neumann series (in powers of λ) involving convolutions of such kernels; (iii) almost degenerate kernels, which are sums of those in the preceding two cases; (iv) uniformly continuous kernels, which by the Weierstrass Theorem can be uniformly approximated with arbitrary accuracy by a degenerate kernel, i.e., a polynomial; and, finally, (v) kernels of the type $K(P, Q)/PQ^\alpha$, where $K(P, Q)$ is uniformly continuous, PQ is the distance between P and Q and $0 \leqq \alpha < d$.

Volterra equations are noted to be a special case of (v) and are dismissed with a brief discussion.

The remaining half of the book is devoted to integral equations with real symmetric kernels and to eigenfunction expansions of their kernels and solutions. Riemann integration and piecewise continuous kernels are assumed, but an appendix is given at the end of the book indicating modifications needed to handle equations with kernels that are square integrable in the Lebesgue sense. Analogies between n-dimensional Euclidean space and function spaces in which the integral equation is investigated are emphasized by parallel columns of