CONVERGENCE OF STOCHASTIC PROCESSES

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1. Many problems in probability theory, when properly formulated, appear as problems in the theory of convergence of stochastic processes. The need for such a theory was demonstrated by the early results of Doob [4], Donsker [5] and others. In their fundamental papers, LeCam [10] and Prohorov [11] developed several aspects of such a theory. Their work was based on, and was a development of, the earlier work of A. D. Alexandrov [1] and Kolmogorov [9]. However, several questions which naturally arise were either not discussed or discussed only under unnecessary restrictions. The following remarks contain an outline of a general theory of measures on topological spaces. Only the statements and the appropriate formulations of the main results are given. The detailed proofs will be published elsewhere.

2. Let X be a topological space and $C(X)$ the Banach space of bounded real-valued continuous functions on X. S is the smallest σ-field of subsets of X with respect to which all the elements of $C(X)$ are measurable. By measure we mean probability measures defined on S and these arise, in the classical manner following F. Riesz, from linear functionals ϕ defined on $C(X)$. Given a nonnegative linear functional ϕ on $C(X)$ with $\phi(1)=1$, we have the representation

$$\phi(f) = \int_X f d\mu \tag{1}$$

for all $f \in C(X)$ with a (unique) measure μ, provided ϕ is *σ-smooth*, i.e. for any sequence $\{f_n\}$ of elements of $C(X)$, $\downarrow 0$ pointwise over X, $\phi(f_n) \to 0$. The set of all measures is denoted by $M(X)$, or simply by M, when there is no doubt as to what X is.

M is a subset of the dual-space of $C(X)$ and as such inherits the weak topology of the dual of $C(X)$. Our main concern is with the structure of this topology over M and its subsets. The two main problems examined are the metrizability of M and the structure of compact subsets of M.

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[1] This work was done during 1958–1959 while the author was in the Indian Statistical Institute, Calcutta, but due to diverse reasons the announcement was delayed up to now.