FOURIER EXPANSIONS OF ARITHMETICAL FUNCTIONS

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Let $f(n)$, $g(n)$ denote complex-valued arithmetical functions with

$$f(n) = \sum_{d|n} g(d). \tag{1}$$

Wintner has proved [3, §33] that if

$$\sum_{n=1}^{\infty} \frac{g(n)}{n} \tag{2}$$

converges absolutely, then $f(n)$ is almost periodic (B) with the absolutely convergent Fourier expansion,

$$f(n) \sim \sum_{r=1}^{\infty} a_r c(n, r), \qquad a_r = \sum_{n=1;\, r|n}^{\infty} \frac{g(n)}{n}, \tag{3}$$

$c(n, r)$ denoting Ramanujan's trigonometric sum. In addition, Wintner showed [3, §35] that if

$$\sum_{n=1}^{\infty} \frac{\tau(n) \, | g(n) |}{n} \tag{4}$$

is convergent, where $\tau(n)$ denotes the number of divisors of n, then $f(n)$ is represented for all n by its Fourier series,

$$f(n) = \sum_{r=1}^{\infty} a_r c(n, r), \qquad a_r = \sum_{n=1;\, r|n}^{\infty} \frac{g(n)}{n}. \tag{5}$$

In this announcement we point out that for certain important classes of multiplicative functions $f(n)$ for which (2) is absolutely convergent (including many such examples that are familiar), the convergence of (4) is not needed to ensure the validity of (5). In fact, we have the following result.

THEOREM I. *Suppose that $f(n)$ is multiplicative and that* (2) *converges absolutely. In case either*

(i) *$g(n)$ is completely multiplicative,*

or in case

(iia) *$g(n) = 0$ when n is not square-free, and*

(iib) *$g(p) \neq -p$ for all primes p,*

then (5) *holds with the convergence absolute.*