ON A HOMOMORPHISM BETWEEN GENERALIZED GROUP ALGEBRAS

BY ALVIN HAUSNER

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If $G=\{a, b, \cdots\}$ is a locally compact abelian group and $X=\{x, y, \cdots\}$ a complex commutative Banach algebra, we denote by $B(G, X)$ the generalized group algebra in the sense of [**1**; **2**]. An X-valued function g defined over G is in $B(G, X)$ if g is strongly measurable and Bochner integrable with respect to Haar measure over G. We define $\|g\|_{B(G,X)}=\int_G |g(a)|_X da$ and, with convolution as multiplication, $B(G, X)$ is a complex commutative B-algebra. In [**1**, p. 1606], it is shown that the space $\mathfrak{M}(B)$ of regular maximal ideals in $B(G, X)$ is homeomorphic with $\hat{G}\times\mathfrak{M}(X)$. Here, $\hat{G}=\{\hat{a}, \hat{b}, \cdots\}$ is the character group of G and $\mathfrak{M}(X)$ denotes the space of regular maximal ideals in X, both in their usual topologies. If ϕ_M is the canonical homomorphism of X onto the complex numbers associated with an $M\in\mathfrak{M}(X)$, then a function $g\in B(G, X)$ is represented on $\mathfrak{M}(B)$ by the function $\hat{g}(\hat{a}, M)=\int_G\phi_M g(a)(a, \hat{a})da$, [**1**, p. 1604]. If $f\in L(G)$, $x\in X$, then fx shall denote the function $(fx)(a)=f(a)x$ almost everywhere over G. Clearly $fx\in B(G, X)$. Further, finite linear combinations of functions of the type fx with $f\in L(G)$, $x\in X$ are dense in $B(G, X)$.

In this paper we propose to characterize the homomorphisms T from $B(G, X)$ into $B(G, X')$ which are such that T keeps $L(G)$ "pointwise invariant." More precise statements will be found in the theorems below.

We begin with

THEOREM 1. *Let G be a group such that $\hat{G}$ is connected and let X and X' be commutative B-algebras with identities e, e' respectively. Suppose $\mathfrak{M}(X)$ is totally disconnected and X' is semi-simple. Let T: $B(G, X)\to B(G, X')$ be a continuous homomorphism such that $T(fe)=fe'$ for any $f\in L(G)$. Then there exists a continuous homomorphism σ: $X\to X'$ such that $(Tg)(a)=\sigma g(a)$ for any $g\in B(G, X)$.*

PROOF.[1] If $g'\in B(G, X')$ and g' is represented on its space of maximal ideals $\hat{G}\times\mathfrak{M}(X')$ as $\hat{f}\cdot\phi'$ where $f\in L(G)$ and ϕ' is a function defined on $\mathfrak{M}(X')$, then $g'=fx'$ for some $x'\in X'$. (Here, $\hat{f}(\hat{a})=\int_G f(\hat{a})(a, \hat{a})da$.) For, consider the function F from G to X' given

[1] The author wishes to gratefully thank the referee of [1] for suggesting the following proof.