RESEARCH ANNOUNCEMENTS

The purpose of this department is to provide early announcement of significant new results, with some indications of proof. Although ordinarily a research announcement should be a brief summary of a paper to be published in full elsewhere, papers giving complete proofs of results of exceptional interest are also solicited.

A NEW TOPOLOGY FOR VON NEUMANN ALGEBRAS

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1. **Introduction.** The theory of von Neumann (v.N.) algebras is rich with topological structure, with many algebraic theorems being proved through the interplay of the various topologies. The strong and strongest topologies (von Neumann [7]) have the properties:

(1) The maps $a \to ab$ and $a \to ba$ are continuous.

(2) The map $(a, b) \to ab$ is continuous for $\|a\| \leqq 1$. The weak and ultra-weak topologies (von Neumann [7]; Dixmier [2]) have property (1) and the following property:

(3) The map $a \to a^*$ is continuous.

The failure of property (2) for the ultra-weak topology and the failure of property (3) for the strongest topology cause much of the difficulties in handling these topologies. In this paper a new topology is introduced which has the properties (1)–(3) and also yields the same continuous linear functionals as the ultra-weak (hence also the strongest) topology. Using this topology we obtain a simpler proof of the Kaplansky Density Theorem (Kaplansky [3]). We also obtain new proofs of a pair of theorems due to Dixmier [2].

Throughout this paper we use a result due to Sakai [6], which states that a B^*-algebra $\mathfrak{A}$ has a representation as a v.N. algebra iff as a Banach Space $\mathfrak{A}$ is the adjoint of a Banach space X. We also obtain a theorem which shows that X is determined up to equivalence by $\mathfrak{A}$.

2. **The $\mu(\mathfrak{A}, X)$ topology.** Let X be a Banach space, $\mathfrak{A}$ its adjoint space and suppose that $\mathfrak{A}$ is a B^*-algebra. We denote the weak topology on X by $\sigma(X, \mathfrak{A})$ and the weak $*$-topology on $\mathfrak{A}$ by $\sigma(\mathfrak{A}, X)$. Sakai [6] shows that properties (1) and (3) hold for the $\sigma(\mathfrak{A}, X)$ topology. We define the mappings $a \to R_a$ and $a \to L_a$ of $\mathfrak{A}$ into $B(X)$ by $R_a x(b) = x(ba)$ and $L_a x(b) = x(ab)$. The map $a \to R_a (a \to L_a)$ is an

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