$H^*(S^{2r}\times S^{2s}; Z_p)$ where $2r$ and $2s$ are even and positive and $r\leqq m$, $s\leqq n$.
Furthermore, if $r\neq s$, then $H^*(X^\pi; Z_p)$ is isomorphic to

$$H^*(S^{2r}\times S^{2s}; Z_p)$$

as a ring.

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A COMPLETE CHARACTERIZATION FOR EXTREME FUNCTIONALS

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1. **Introduction.** Let E be a real linear space with a seminorm $\|\ \|$. Let S be the set of linear functionals of norm less than or equal to 1; thus, $L\in S$ whenever $|L(x)|\leqq\|x\|$ for all $x\in E$. In many problems, it is important to know something about the extreme points of S; L is extreme if we cannot write $L=(L'+L'')/2$ with L' and L'' distinct members of S. In this note, a new procedure will be developed for the study of a particular functional L; in particular, this will provide a surprisingly simple and useful characterization for the extreme functionals. With each L, we shall associate a closed subspace V_L of E in such a fashion that the relative "flatness" of S at L is determined by the size of V_L. In particular, L is an extreme point of S if and only if V_L is all of E. The results are formulated for a *seminormed* space E to allow their application to certain special cases of considerable interest. One such question is discussed in the last section where we look for extreme functionals in the class of those that vanish on a fixed subspace M.

2. **Construction of V_L.** An equivalent way to say that L is extreme in S is to say that a functional θ obeys $\|L\pm\theta\|\leqq 1$ only if $\theta=0$. Because we are dealing with a real space, this condition on θ can be rewritten as

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